SpECTRE: Towards improved simulations of relativistic astrophysical systems

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1 Background and motivation

2 Numerical methods

3 SpECTRE implementation
Simulations of GRMHD coupled to Einstein’s equations are complicated, difficult, and interesting
Simulation Goals

- Accretion disks
- Binary neutron star mergers
- Core-collapse supernova explosions

Event Horizon Telescope Collaboration
Need For High Accuracy

- Gravitational waveforms for LIGO/Virgo and space-based detectors
- LIGO/Virgo follow-up waveforms
- Accretion for Event Horizon Telescope
- Improved understanding of heavy element generation

Abbott et al. 2017

github.com/sxs-collaboration/spectre
• Hyperbolic equations in general form:

\[ \partial_t U + \partial_i F^i(U) + B^i \cdot \partial_i U = S(U) \]

• Elliptic equations of the form:

\[ \partial^2 U = S(U, \partial U) \]
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Vacuum Evolutions: Spectral Methods

- Smooth solutions
- Exponential convergence
- Non-overlapping grids
- General grids:

[Images of grid structures]
Hydrodynamics: Finite Volume Methods

- Work on shocks
- Polynomial convergence

- Typically Cartesian grids
- Overlapping grids

$u(x)$

\[
\text{github.com/sxs-collaboration/spectre}
\]
Current codes:

- Message passing (MPI) + some threading
- Spectral Einstein Code (SpEC):
  - Spectral methods: one element per core
  - Finite volume: $\sim 100,000 - 150,000$ cells per core
- Pseudospectral methods $\sim 50$ cores
- Finite volume methods $\sim 20,000$ cores
• Exponential convergence for smooth solutions
• Shock capturing
• Non-overlapping deformed grids
• $hp$-adaptivity
• Local time stepping
• Nearest-neighbor communication
- Boundary fluxes communicated between elements
- Nearest-neighbor only, good for parallelization
• Consider element $\Omega_{k-1}$:

$$G_{k-1} = \frac{1}{2} \left( F_{i,+}^{i,n_i^+} + F_{i,-}^{i,n_i^-} \right) - \frac{C}{2} (u^+ - u^-)$$
The DG Algorithm Summary

1. Compute time derivatives
2. Send data for boundary data
3. Integrate in time
4. Send data for limiting
5. Apply limiter
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SpECTRE Design Goals

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• General framework for hyperbolic (Cornell, Caltech, CalState Fullerton, UNH) and elliptic (AEI) PDEs
Available Physical Systems

- Scalar wave
- Curved scalar wave (mostly)
- Newtonian Euler (in code review)
- Relativistic Euler (mostly)
- GRMHD
- Generalized harmonic (in code review)
Numerical Schemes

Numerical fluxes:
- Rusanov (local Lax-Friedrichs)
- HLL
- Upwind

Planned numerical fluxes:
- HLLC
- Roe
- Marquina

Limiters:
- Minmod (MUSCL, $\Lambda^{1}$, $\Lambda^{N}$)
- Krivodonova
- SimpleWENO (in code review)
- HWENO (in code review)
- Multipatch FV/FD subcell (in progress)

Planned limiters:
- Moe-Rossmanith-Seal (MRS)
- Hierarchical Barth-Jespersen and vertex-based
Convergence for Smooth Problems: Alfvén Wave

\[ L_1(\varepsilon(v_z)) \]

\[ N_x = 4 \]
\[ N_x = 8 \]
\[ N_x = 16 \]
\[ N_x = 32 \]

\[ \Delta x^{-4} \]
\[ \Delta x^{-6} \]
\[ \Delta x^{-8} \]
\[ \Delta x^{-10} \]

\[ P_1 \]
\[ P_3 \]
\[ P_5 \]
\[ P_7 \]
\[ P_9 \]
\[ P_{11} \]
Single Black Hole Evolutions

- Generalized harmonic system
- Excised cube in center

\[ L_2(\mathcal{H} + \Gamma_a) \]
\[ \text{Error}(\Phi_{iab}) \]
\[ \text{Error}(g_{ab}) \]
\[ \text{Error}(\Pi_{ab}) \]
256 × 1 × 1 elements, $3^3$ points per element

Krivodonova

SimpleWENO

HWENOW
128^2 \times 1 \text{ elements, } 2^3 \text{ points per element}
Cylindrical Blast Wave

$128^2 \times 1$ elements, $3^3$ points per element

Krivodonova

SimpleWENO

HWENO

github.com/sxs-collaboration/spectre
• Torus around a black hole
• Code comparison project
• $\chi = 0.9375$, $\rho_{\text{max}} \approx 77$
• Orbital period $T_{\text{orb}} \approx 247$
• Hexahedron: $[-40, 40] \times [2, 40] \times [-8, 8]$
Rest mass density $\rho$ at $t = 600$
Error in rest mass density $\rho$ at $t = 600$
- Run on BlueWaters supercomputer, NCSA, UIUC, IL, USA
- Green is perfect speedup for fixed problem size (strong scaling)
- Blue shows actual weak scaling (flat is ideal)
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• Limiting and primitive recovery an open problem