SpECTRE: Towards improved simulations of relativistic astrophysical systems

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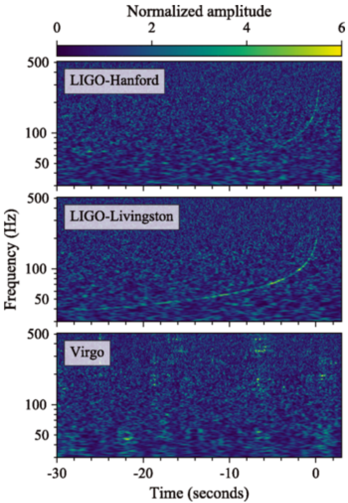
Simulations of GRMHD coupled to Einstein's equations are complicated, difficult, and interesting

- Accretion disks
- Binary neutron star mergers
- Core-collapse supernova explosions

Event Horizon Telescope Collaboration

- Gravitational waveforms for LIGO/Virgo and space-based detectors
- LIGO/Virgo follow-up waveforms
- Accretion for Event Horizon Telescope
- Improved understanding of heavy element generation



Abbott et al. 2017

- Hyperbolic equations in general form:

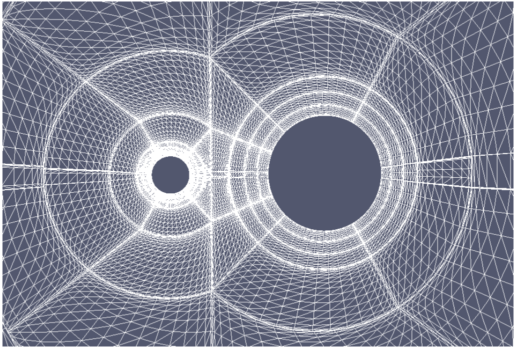
$$\partial_t \mathbf{U} + \partial_i \mathbf{F}^i(\mathbf{U}) + \mathbf{B}^i \cdot \partial_i \mathbf{U} = \mathbf{S}(\mathbf{U})$$

- Elliptic equations of the form:

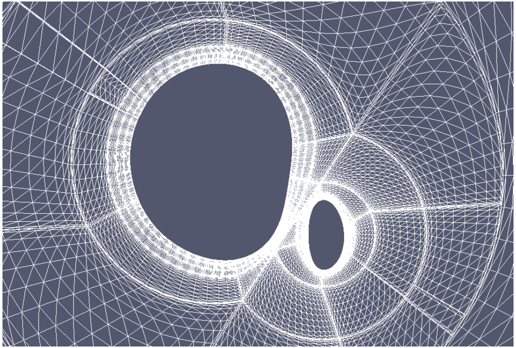
$$\partial^2 \mathbf{U} = \mathbf{S}(\mathbf{U}, \partial \mathbf{U})$$

Vacuum Evolutions: Spectral Methods

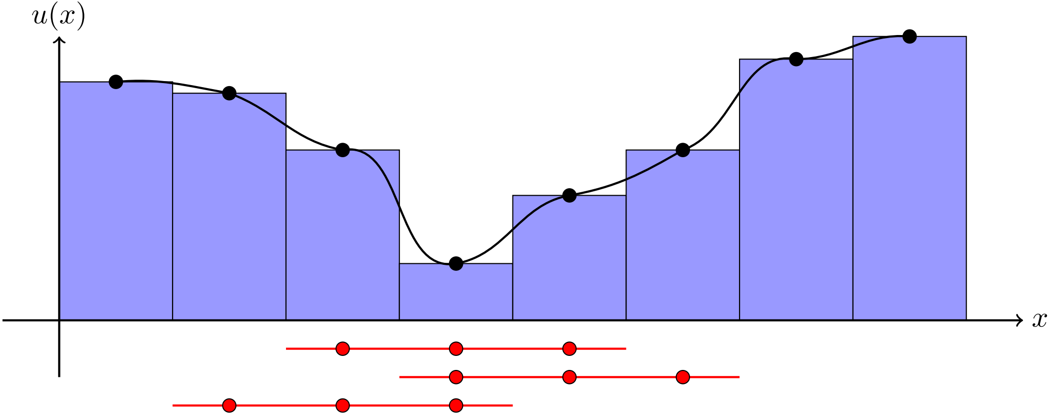
- Smooth solutions
- Exponential convergence



- Non-overlapping grids
- General grids:



- Work on shocks
- Polynomial convergence
- Typically Cartesian grids
- Overlapping grids

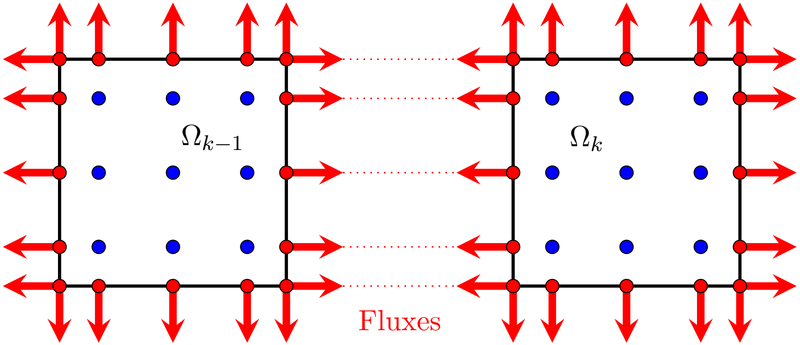


Current codes:

- Message passing (MPI) + some threading
- Spectral Einstein Code (SpEC):
  - Spectral methods: one element per core
  - Finite volume: $\sim 100,000 - 150,000$ cells per core
- Pseudospectral methods ~ 50 cores
- Finite volume methods $\sim 20,000$ cores

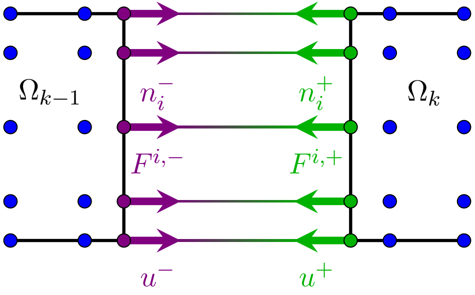
- Exponential convergence for smooth solutions
- Shock capturing
- Non-overlapping deformed grids
- *hp*-adaptivity
- Local time stepping
- Nearest-neighbor communication

- Boundary fluxes communicated between elements
- Nearest-neighbor only, good for parallelization



- Consider element Ω_{k-1}:

$$G_{k-1} = \frac{1}{2}\left(F^{i,+}n_i^+ + F^{i,-}n_i^-\right) - \frac{C}{2}\left(u^+ - u^-\right)$$



1. Compute time derivatives
2. Send data for boundary data
3. Integrate in time
4. Send data for limiting
5. Apply limiter

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SpECTRE Design Goals

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- General framework for hyperbolic (Cornell, Caltech, CalState Fullerton, UNH) and elliptic (AEI) PDEs

Available Physical Systems

- Scalar wave
- Curved scalar wave (mostly)
- Newtonian Euler (in code review)
- Relativistic Euler (mostly)
- GRMHD
- Generalized harmonic (in code review)

Numerical fluxes:

- Rusanov (local Lax-Friedrichs)
- HLL
- Upwind

Planned numerical fluxes:

- HLLC
- Roe
- Marquina

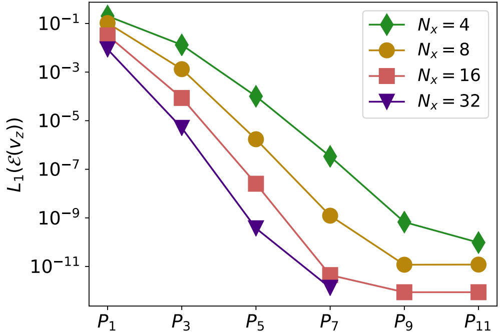
Limiters:

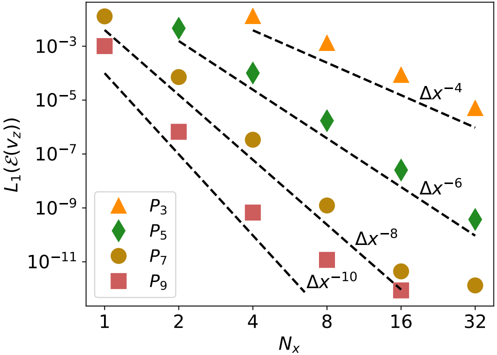
- Minmod (MUSCL, $\Lambda\Pi^1$, $\Lambda\Pi^N$)
- Krivodonova
- SimpleWENO (in code review)
- HWENO (in code review)
- Multipatch FV/FD subcell (in progress)

Planned limiters:

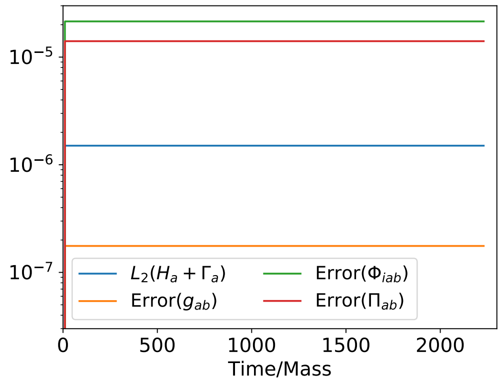
- Moe-Rossmanith-Seal (MRS)
- Hierarchical Barth-Jespersen and vertex-based

Convergence for Smooth Problems: Alfvén Wave

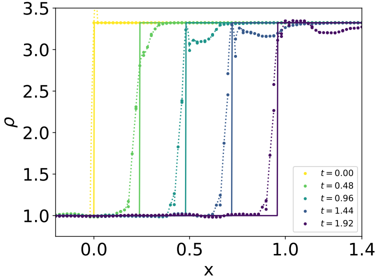




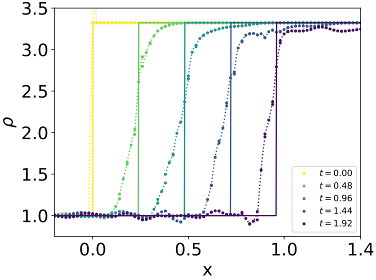
- Generalized harmonic system
- Excised cube in center



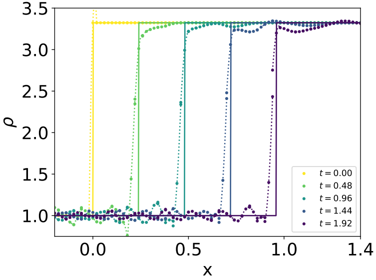
$256 \times 1 \times 1$ elements, 3^3 points per element



Krivodonova

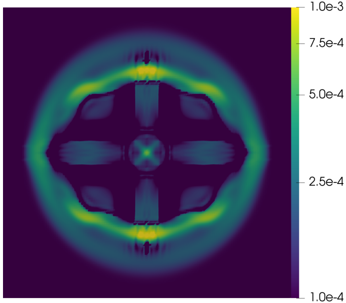


SimpleWENO



HWENO

$128^2 \times 1$ elements, 2^3 points per element



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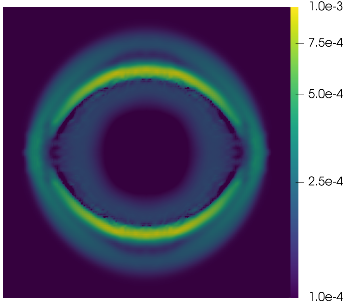
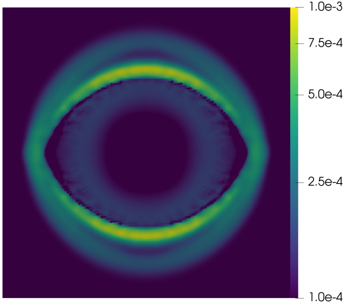
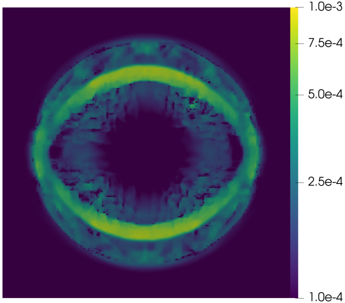


SimpleWENO



HWENO

$128^2 \times 1$ elements, 3^3 points per element



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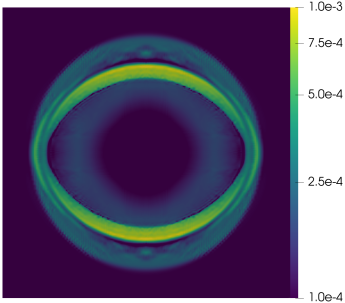

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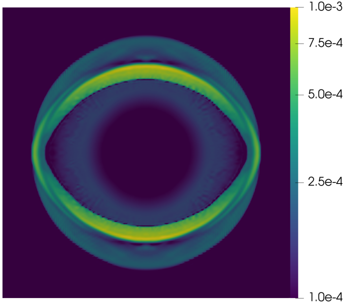
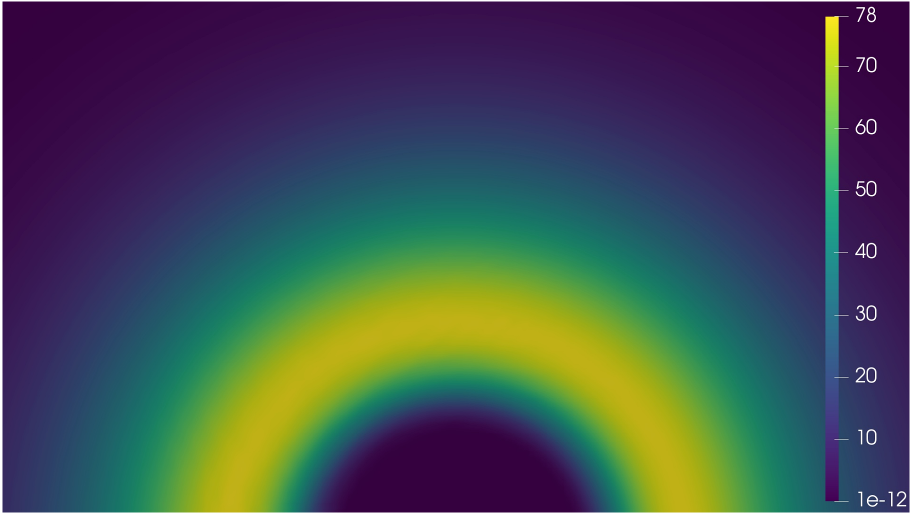


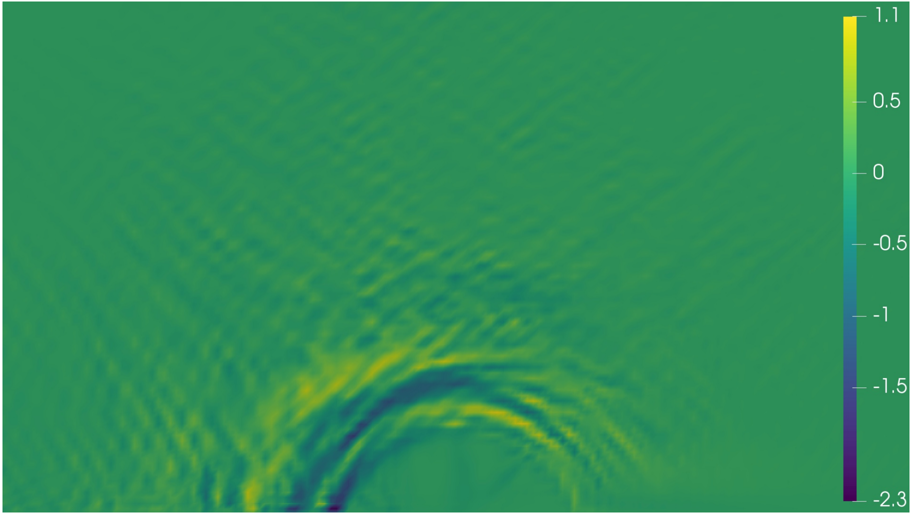
HWENO

- Torus around a black hole
- Code comparison project
- $\chi = 0.9375$, $\rho_{\max} \approx 77$
- Orbital period $T_{\text{orb}} \approx 247$
- Hexahedron: $[-40, 40] \times [2, 40] \times [-8, 8]$

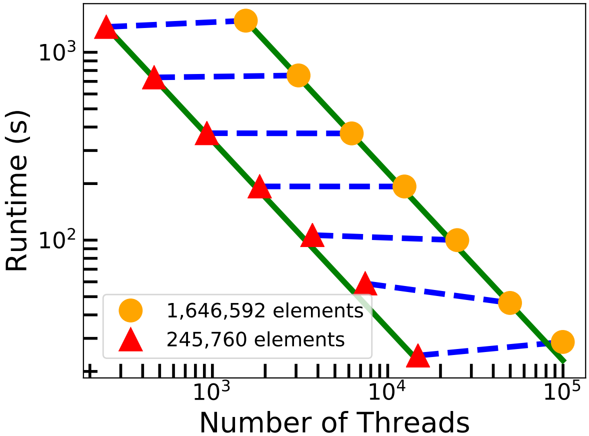
Rest mass density ρ at $t = 600$



Error in rest mass density ρ at $t = 600$



- Run on BlueWaters supercomputer, NCSA, UIUC, IL, USA
- Green is perfect speedup for fixed problem size (strong scaling)
- Blue shows actual weak scaling (flat is ideal)



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Summary

- Improved vacuum and GRMHD simulations necessary for experiment
- Current methods difficult to scale to new machines
- Discontinuous Galerkin as alternative new method
- SpECTRE as general hyperbolic and elliptic PDE solver (not just DG)
- Successful scaling to largest machines available

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- SpECTRE as general hyperbolic and elliptic PDE solver (not just DG)
- Successful scaling to largest machines available
- Limiting and primitive recovery an open problem