



Architecture-Aware Algorithms and Software for Peta and Exascale Computing

Jack Dongarra

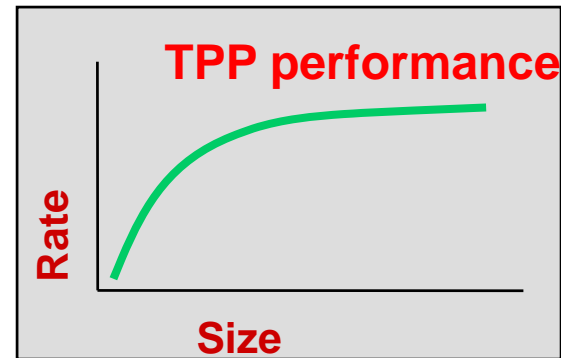
University of Tennessee
Oak Ridge National Laboratory
University of Manchester



H. Meuer, H. Simon, E. Strohmaier, & JD

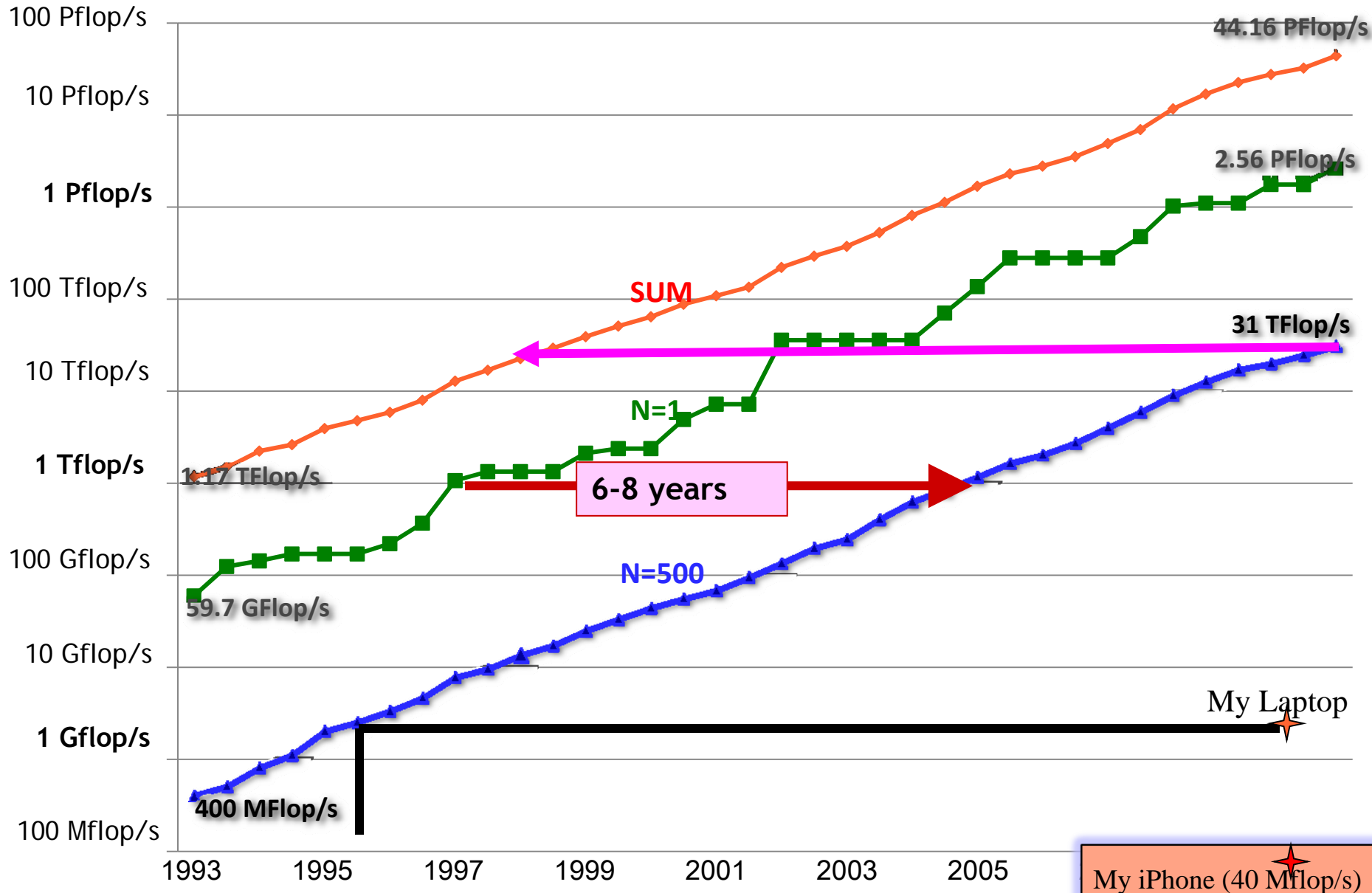
- Listing of the 500 most powerful Computers in the World
- Yardstick: Rmax from LINPACK MPP

$$Ax=b, \text{ dense problem}$$



- Updated twice a year
SC'xy in the States in November
Meeting in Germany in June
- All data available from [www².top500.org](http://www.top500.org)

Performance Development



36rd List: The TOP10

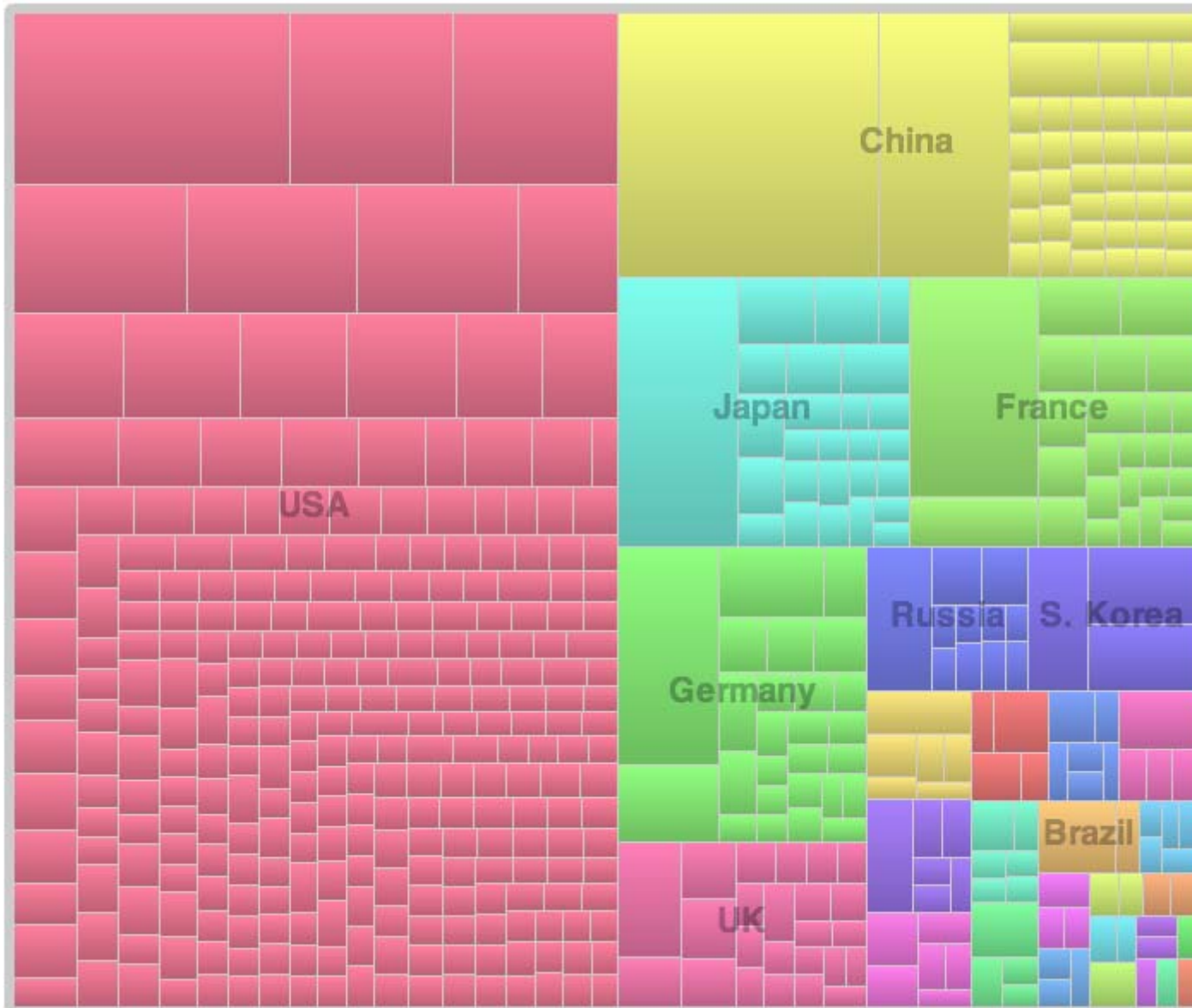
Rank	Site	Computer	Country	Cores	Rmax [Pflops]	% of Peak
1	Nat. SuperComputer Center in Tianjin	Tianhe-1A, NUDT Intel + Nvidia GPU + custom	China	186,368	2.57	55
2	DOE / OS Oak Ridge Nat Lab	Jaguar, Cray AMD + custom	USA	224,162	1.76	75
3	Nat. Supercomputer Center in Shenzhen	Nebulea, Dawning Intel + Nvidia GPU + IB	China	120,640	1.27	43
4	GSIC Center, Tokyo Institute of Technology	Tusbame 2.0, HP Intel + Nvidia GPU + IB	Japan	73,278	1.19	52
5	DOE / OS Lawrence Berkeley Nat Lab	Hopper, Cray AMD + custom	USA	153,408	1.054	82
6	Commissariat a l'Energie Atomique (CEA)	Tera-10, Bull Intel + IB	France	138,368	1.050	84
7	DOE / NNSA Los Alamos Nat Lab	Roadrunner, IBM AMD + Cell GPU + IB	USA	122,400	1.04	76
8	NSF / NICS U of Tennessee	Kraken, Cray AMD + custom	USA	98,928	.831	81
9	Forschungszentrum Juelich (FZJ)	Jugene, IBM Blue Gene + custom	Germany	294,912	.825	82
10	DOE / NNSA LANL & SNL	Cielo, Cray AMD + custom	USA	107,152	.817	79

36rd List: The TOP10

Rank	Site	Computer	Country	Cores	Rmax [Pflops]	% of Peak	Power [MW]	GFlops/Watt
1	Nat. SuperComputer Center in Tianjin	Tianhe-1A, NUDT Intel + Nvidia GPU + custom	China	186,368	2.57	55	4.04	636
2	DOE / OS Oak Ridge Nat Lab	Jaguar, Cray AMD + custom	USA	224,162	1.76	75	7.0	251
3	Nat. Supercomputer Center in Shenzhen	Nebulea, Dawning Intel + Nvidia GPU + IB	China	120,640	1.27	43	2.58	493
4	GSIC Center, Tokyo Institute of Technology	Tusbame 2.0, HP Intel + Nvidia GPU + IB	Japan	73,278	1.19	52	1.40	850
5	DOE / OS Lawrence Berkeley Nat Lab	Hopper, Cray AMD + custom	USA	153,408	1.054	82	2.91	362
6	Commissariat a l'Energie Atomique (CEA)	Tera-10, Bull Intel + IB	France	138,368	1.050	84	4.59	229
7	DOE / NNSA Los Alamos Nat Lab	Roadrunner, IBM AMD + Cell GPU + IB	USA	122,400	1.04	76	2.35	446
8	NSF / NICS U of Tennessee	Kraken, Cray AMD + custom	USA	98,928	.831	81	3.09	269
9	Forschungszentrum Juelich (FZJ)	Jugene, IBM Blue Gene + custom	Germany	294,912	.825	82	2.26	365
10	DOE / NNSA LANL & SNL	Cielo, Cray AMD + custom	USA	107,152	.817	79	2.95	277

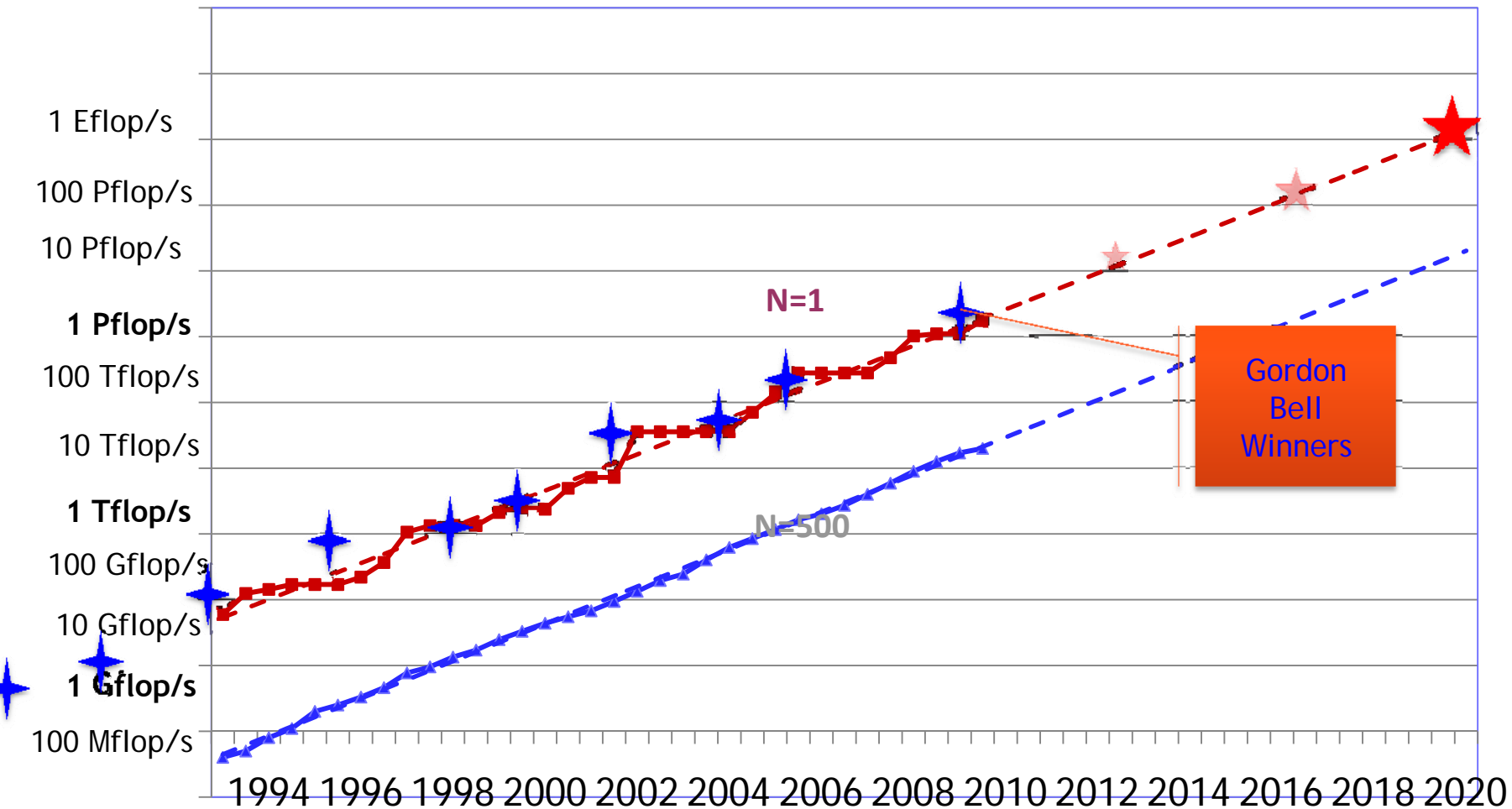
500 Computacenter LTD HP Cluster, Intel + GigE UK 5,856 .031 53

Countries Share



Absolute Counts	
US:	274
China:	41
Germany:	26
Japan:	26
France:	26
UK:	25

Performance Development in Top500





Potential System Architecture

Systems	2010
System peak	2 Pflop/s
Power	6 MW
System memory	0.3 PB
Node performance	125 GF
Node memory BW	25 GB/s
Node concurrency	12
Total Node Interconnect BW	3.5 GB/s
System size (nodes)	18,700
Total concurrency	225,000
Storage	15 PB
IO	0.2 TB
MTTI	days



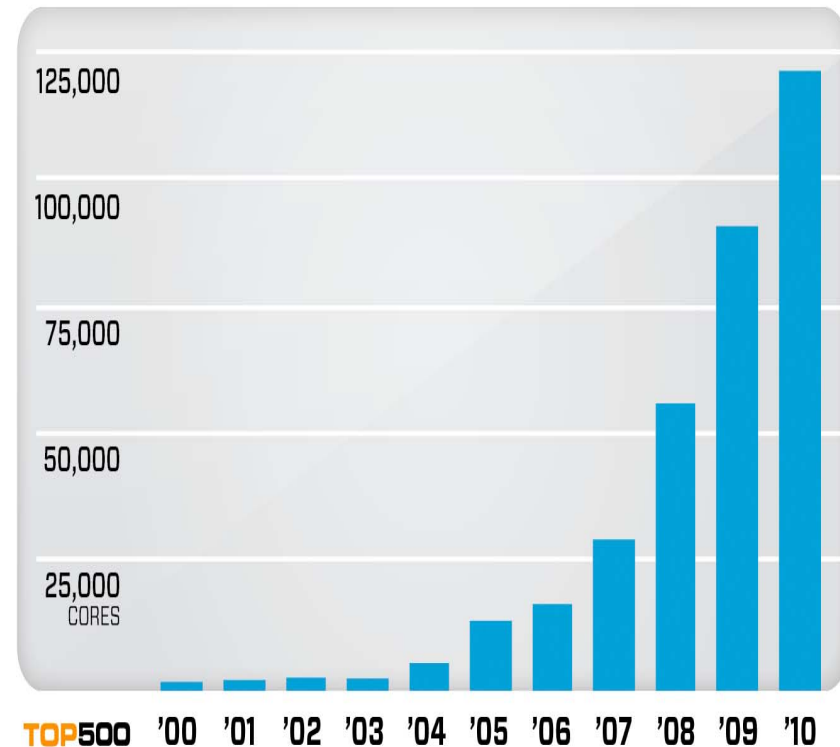
Potential System Architecture with a cap of \$200M and 20MW

Systems	2010	2018	Difference Today & 2018
System peak	2 Pflop/s	1 Eflop/s	O(1000)
Power	6 MW	~20 MW	
System memory	0.3 PB	32 - 64 PB	O(100)
Node performance	125 GF	1,2 or 15TF	O(10) - O(100)
Node memory BW	25 GB/s	2 - 4TB/s	O(100)
Node concurrency	12	O(1k) or 10k	O(100) - O(1000)
Total Node Interconnect BW	3.5 GB/s	200-400GB/s	O(100)
System size (nodes)	18,700	O(100,000) or O(1M)	O(10) - O(100)
Total concurrency	225,000	O(billion)	O(10,000)
Storage	15 PB	500-1000 PB (>10x system memory is min)	O(10) - O(100)
IO	0.2 TB	60 TB/s (how long to drain the machine)	O(100)
MTTI	days	O(1 day)	- O(10)

Factors that Necessitate Redesign of Our Software

- Steepness of the ascent from terascale to petascale to exascale
- Extreme parallelism and hybrid design
 - Preparing for million/billion way parallelism
- Tightening memory/bandwidth bottleneck
 - Limits on power/clock speed implication on multicore
 - Reducing communication will become much more intense
 - Memory per core changes, byte-to-flop ratio will change
- Necessary Fault Tolerance
 - MTTF will drop
 - Checkpoint/restart has limitations
 - shared responsibility

Average Number of Cores per Supercomputer for Top 20 Systems

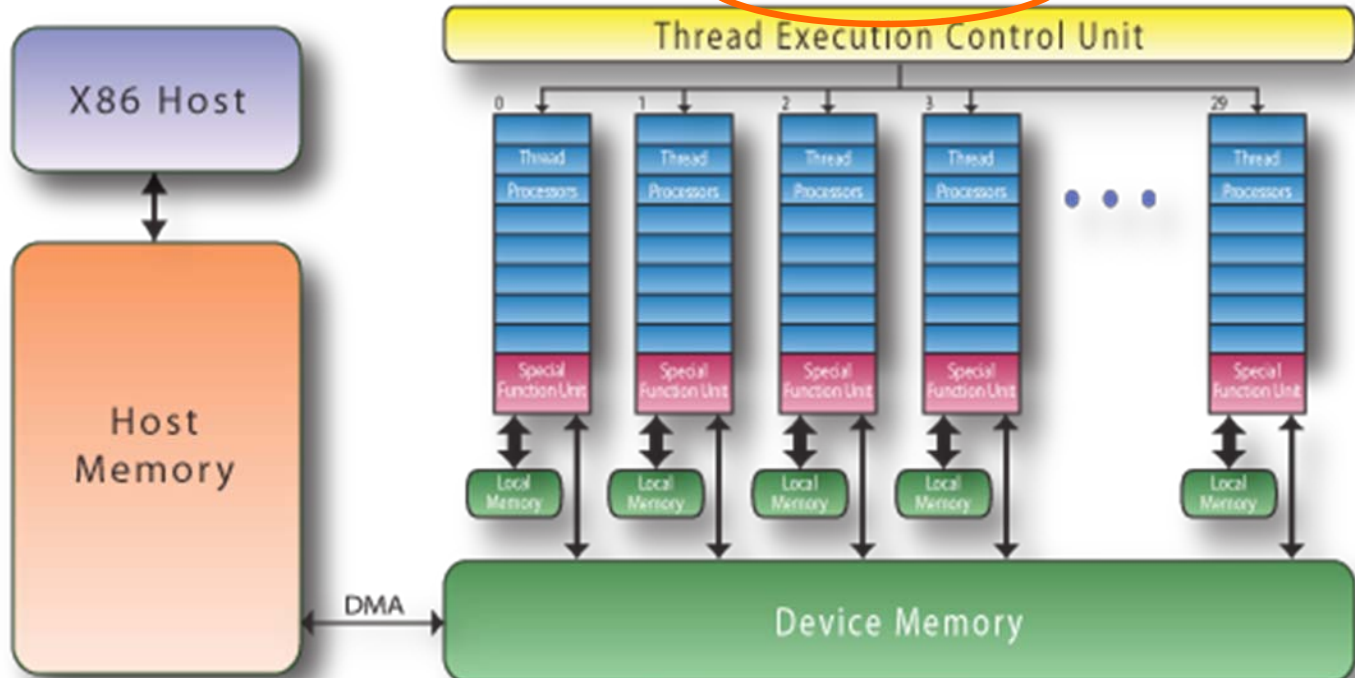


Software infrastructure does not exist today

Commodity plus Accelerators

Commodity
 Intel Xeon
 8 cores
 3 GHz
 8*4 ops/cycle
 96 Gflop/s (DP)

Accelerator (GPU)
 Nvidia C2050 "Fermi"
 448 "Cuda cores"
 1.15 GHz
 448 ops/cycle
 515 Gflop/s (DP)



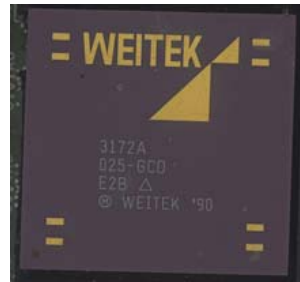
Interconnect
 PCI-X 16 lane

64 Gb/s
 1 GW/s

17 systems on the TOP500 use GPUs as accelerators

We Have Seen This Before

- Floating Point Systems FPS-164/MAX Supercomputer (1976)
- Intel Math Co-processor (1980)
- Weitek Math Co-processor (1981)



THREE HUNDRED FORTY ONE MILLION FLOATING POINT OPERATIONS PER SECOND.

THE FPS-164/MAX.

1976

Radix scientific and engineering problems increasingly call for super-computer strength and higher resolution, which leads to calculations on very large numbers. We built new kind of a super computer with the speed and accuracy needed to solve problems of this nature.

Now, there's the FPS-164/MAX... a special purpose computer system designed that matches the size of CRAY, CYBER, and others in scientific and engineering applications... at a fraction of the cost.

The FPS-164/MAX is fast.
 300 word performance is over three 30 to 300 million floating point operations per second, depending on configuration. And, up to 7 times as fast as currently available in the cost. But new FPS-164/MAX gives you all the speed and accuracy you need to solve those really complicated problems.

The FPS-164/MAX evaluations are done in accordance with D.E. member operations at the time following a fully configured FPS-164/MAX system at 1.00 to 1.000 million in about 1 second, reaching over 300,000 to 30,000,000 in one hour.

The FPS-164/MAX is powerful.
 A world-renowned leader in super-computer systems, FPS-164/MAX has all the super-computer of our original FPS-164. We've just added a lot more power with our super-special processing units which simplify the super-computer sized size of the original FPS-164 by up to 10 times.

The FPS-164/MAX is cost-effective.
 Its modular design, optional displays and graphics, built-in systems, electromagnetic shielding, its easy application requiring few hours of labor, makes the FPS-164/MAX very well-suited for use in many applications. In fact, it's the only super-computer that's so easy to use. It's the only super-computer that's so easy to use. It's the only super-computer that's so easy to use.

Key Features:

- High-Speed Floating Point Operations
- Large Memory Capacity
- High-Speed Input/Output
- Optional Graphics and Displays
- Electromagnetic Shielding
- Easy Application

LOOKING FOR A SYSTEM? IFC

ICL, New 20480
 Fremont, CA 94533
 (415) 841-3535
 ICL, 30000 CALIFORNIA BLVD

Circle Number 338 on Reader Service Card

The Intel® Math CoProcessor™ is for crunching numbers faster.

intel

Personal Computer Enhancement

There's one for every machine.

80387™ Family, For 80387™ based machines.

80387 Family, For 80286-based machines.

80387 Family, For 8086 and 8088 based machines.

80387 Family, For 8088™ and 8088SX™ based machines.

It's FAST!
 The Intel Math CoProcessor dramatically speeds up the number crunching that's part of the work you do every day: budgeting, statistical analysis, financial analysis, CAD and other engineering analysis. In fact, the Math CoProcessor is supported by more than 100 commonly used software packages including Lotus 1-2-3, dBase IV, AutoCAD, and most languages and statistical packages.

It's EASY!
 Intel makes a variety of math coprocessors. Every PC has a built-in socket. Just plug it in and go.

It's SAFE!
 Made by Intel, the same people who designed your PC's microprocessor, each and every Math CoProcessor is backed by an industry leading five year warranty and full free technical support. You are assured the highest degree of quality, compatibility, reliability and support for your investment. For more information or technical support call:

(800) 538-3573 in the US and Canada
 (510) 528-7564 in International

Intel Math CoProcessors are made with the same quality and care as all Intel products. They are backed by the same quality and care as all Intel products.

intel

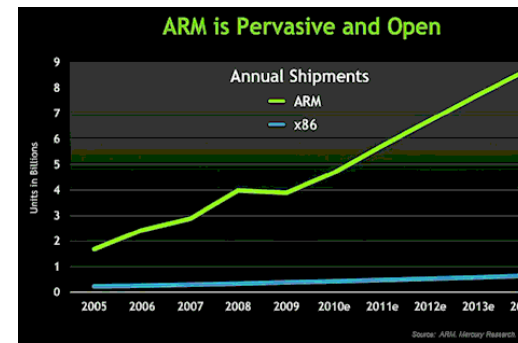
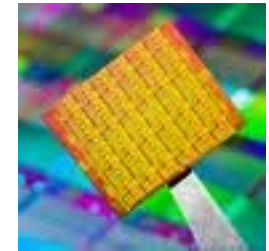
1980

Personal Computer Enhancement

Future Computer Systems



- **Most likely be a hybrid design**
 - **Think standard multicore chips and accelerator (GPUs)**
- Today accelerators are attached
- Next generation more integrated
- Intel's MIC architecture "Knights Ferry" and "Knights Corner" to come.
 - **48 x86 cores**
- **AMD's Fusion in 2012 - 2013**
 - **Multicore with embedded graphics ATI**
- **Nvidia's Project Denver plans to develop an integrated chip using ARM architecture in 2013.**



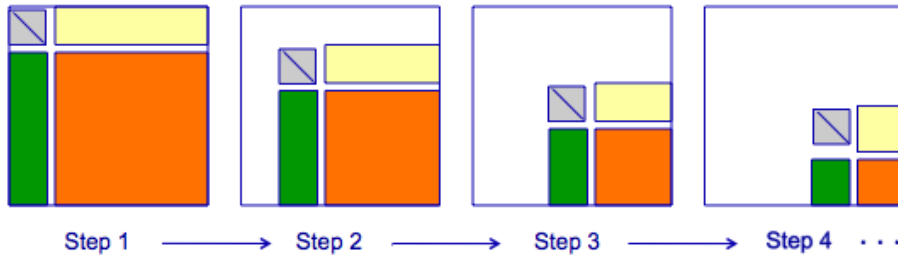
Major Changes to Software

- **Must rethink the design of our software**
 - **Another disruptive technology**
 - Similar to what happened with cluster computing and message passing
 - **Rethink and rewrite the applications, algorithms, and software**

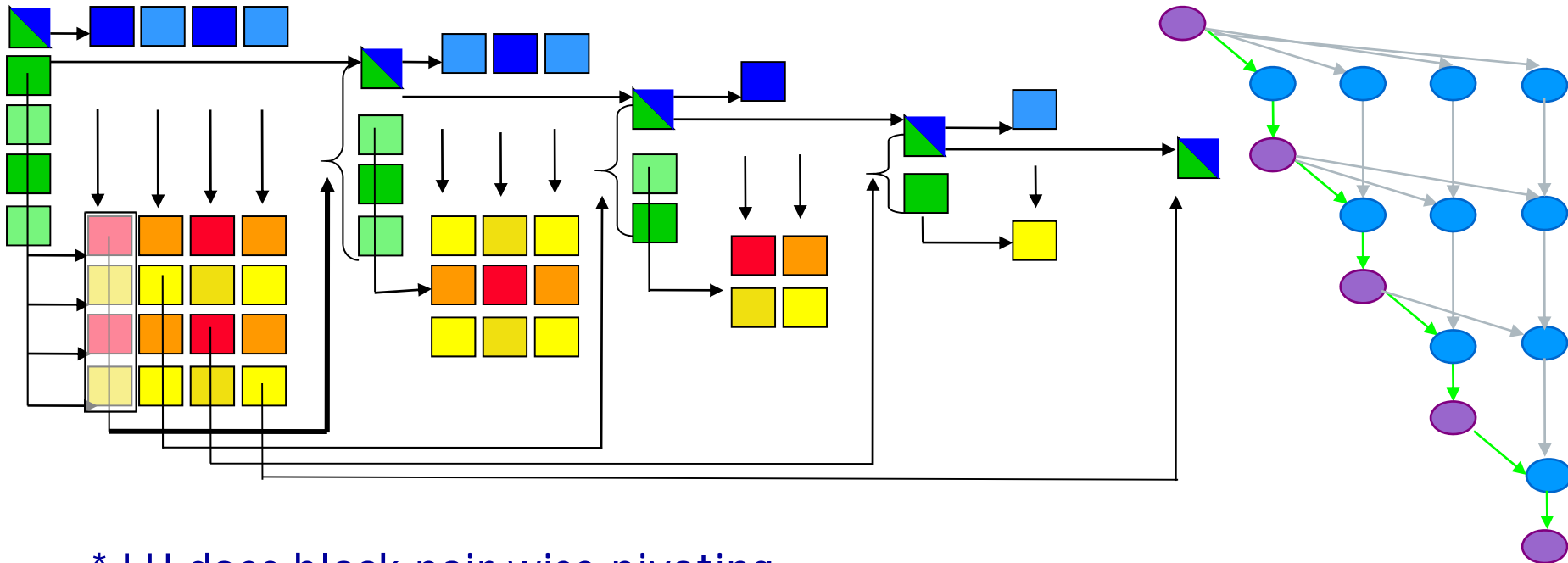
Exascale algorithms that expose and exploit multiple levels of parallelism

- **Synchronization-reducing algorithms**
 - **Break Fork-Join model**
- **Communication-reducing algorithms**
 - **Use methods which have lower bound on communication**
- **Mixed precision methods**
 - **2x speed of ops and 2x speed for data movement**
- **Reproducibility of results**
 - **Today we can't guarantee this**
- **Fault resilient algorithms**
 - **Implement algorithms that can recover from failures**

Parallel Tasks in LU/LL^T/QR



- Break into smaller tasks and remove dependencies



* LU does block pair wise pivoting

PLASMA: Parallel Linear Algebra s/w for Multicore Architectures

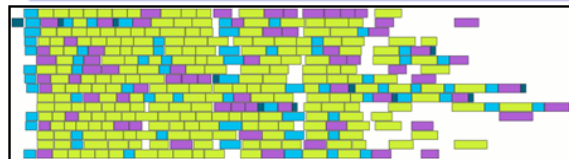
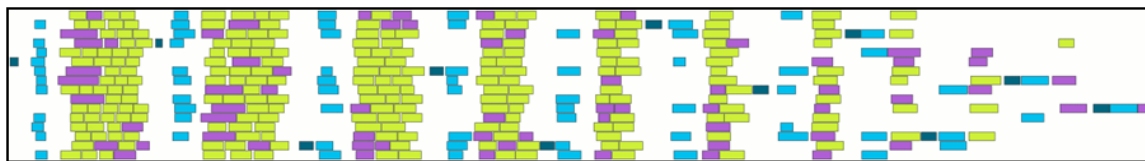
• Objectives

- High utilization of each core
- Scaling to large number of cores
- Shared or distributed memory

• Methodology

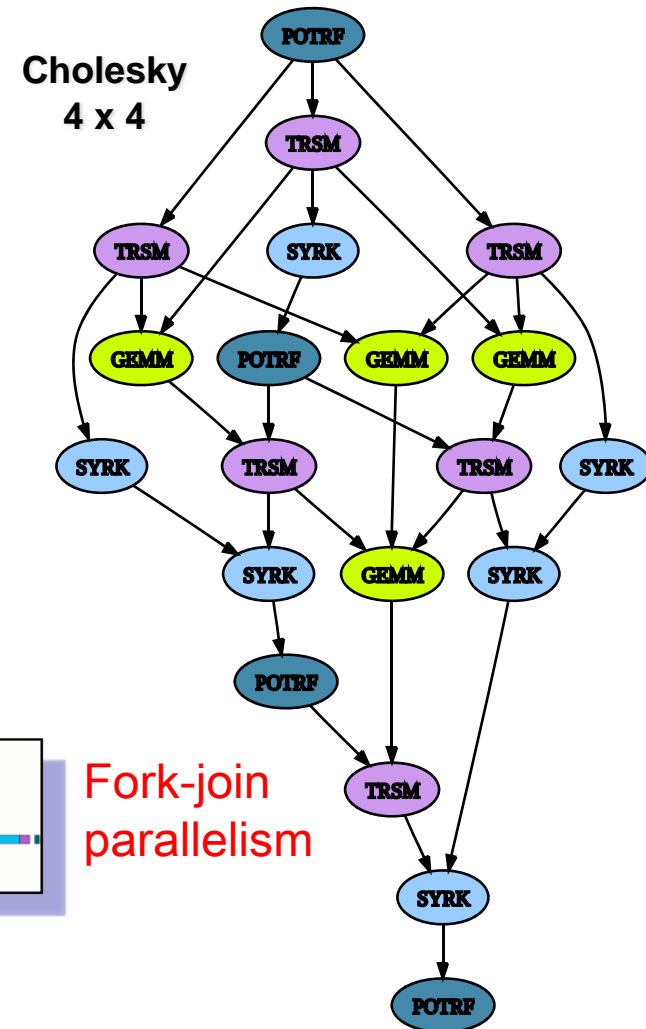
- Dynamic DAG scheduling
- Explicit parallelism
- Implicit communication
- Fine granularity / block data layout

• Arbitrary DAG with dynamic scheduling



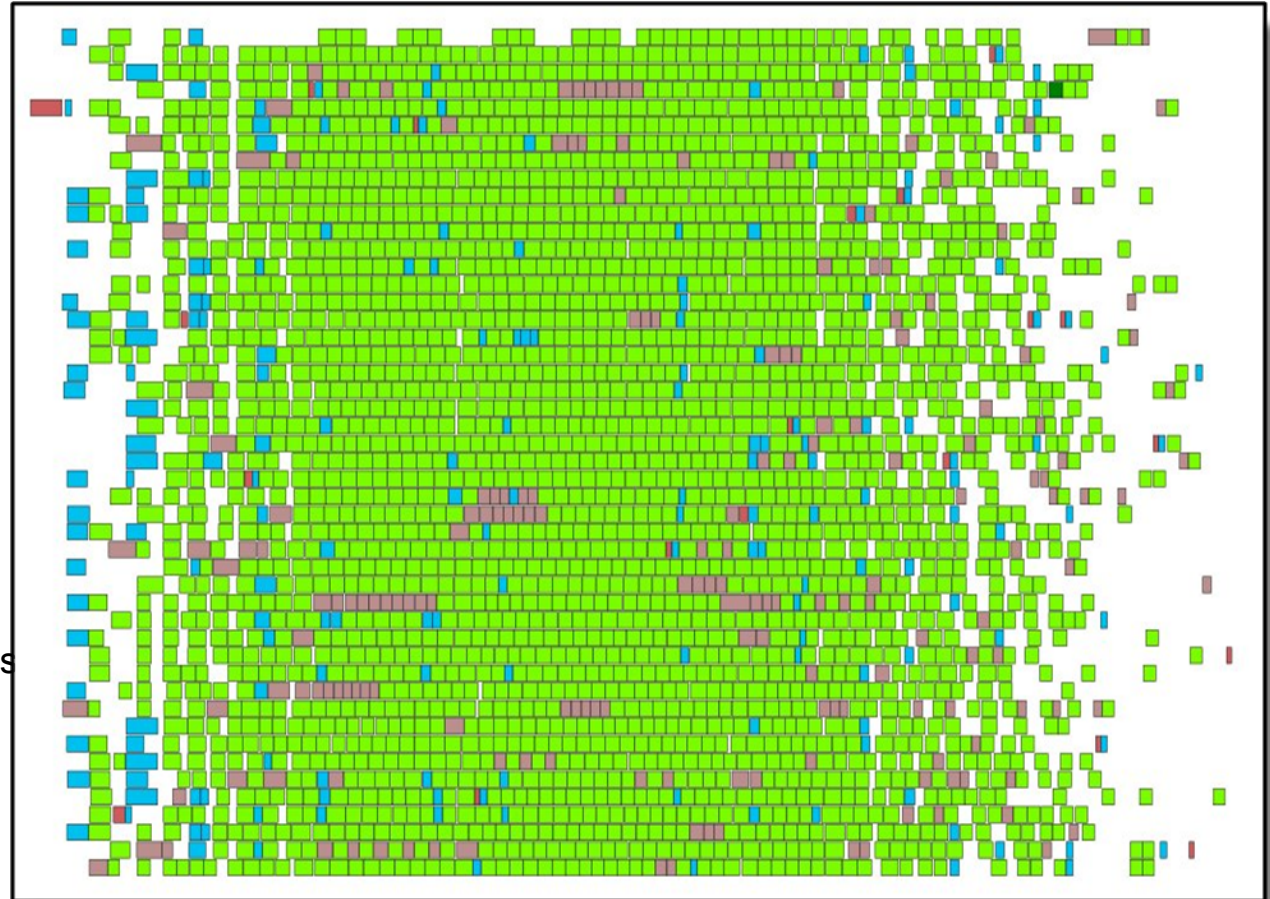
DAG scheduled parallelism

Time



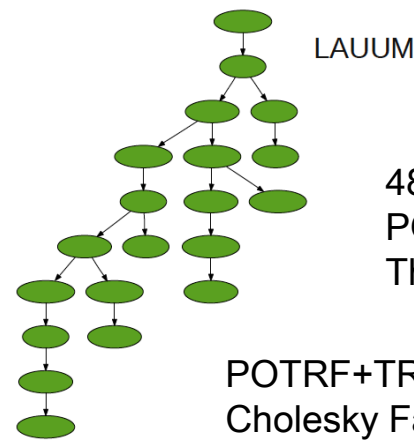
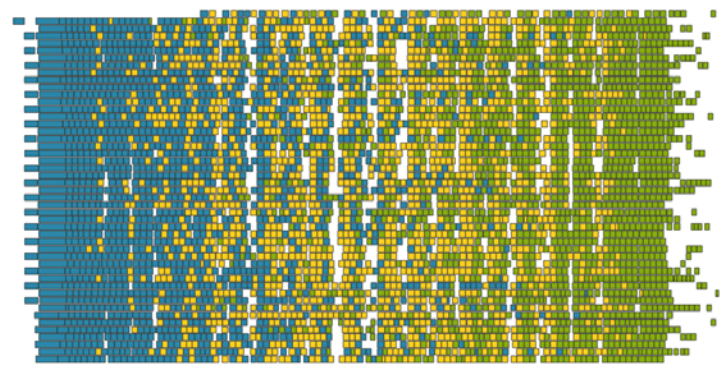
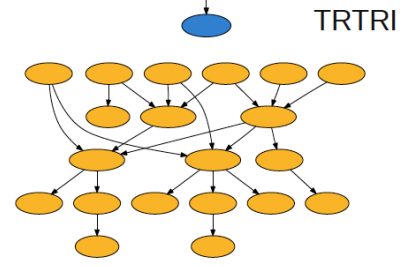
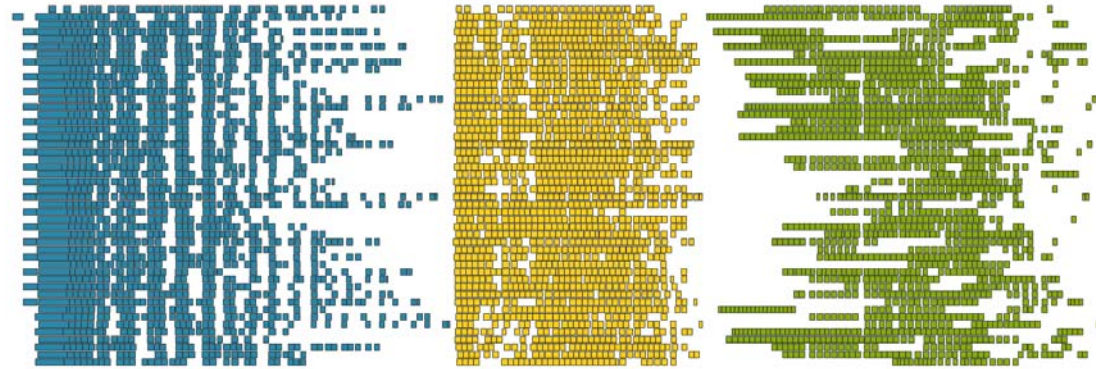
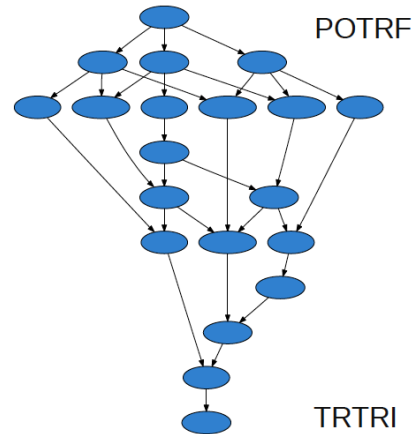
Synchronization Reducing Algorithms

- Regular trace
- Factorization steps pipelined
- Stalling only due to natural load imbalance
- Reduce ideal time
- Dynamic
- Out of order execution
- Fine grain tasks
- Independent block operations



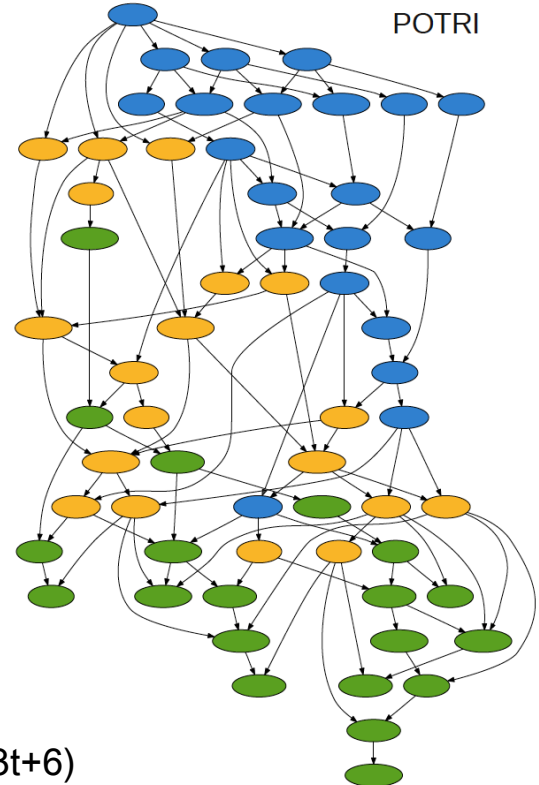
8-socket, 6-core (48 cores total) AMD Istanbul 2.8 GHz

Pipelining: Cholesky Inversion



48 cores
 POTRF, TRTRI and LAUUM.
 The matrix is 4000 x 4000, tile size is 200 x 200,

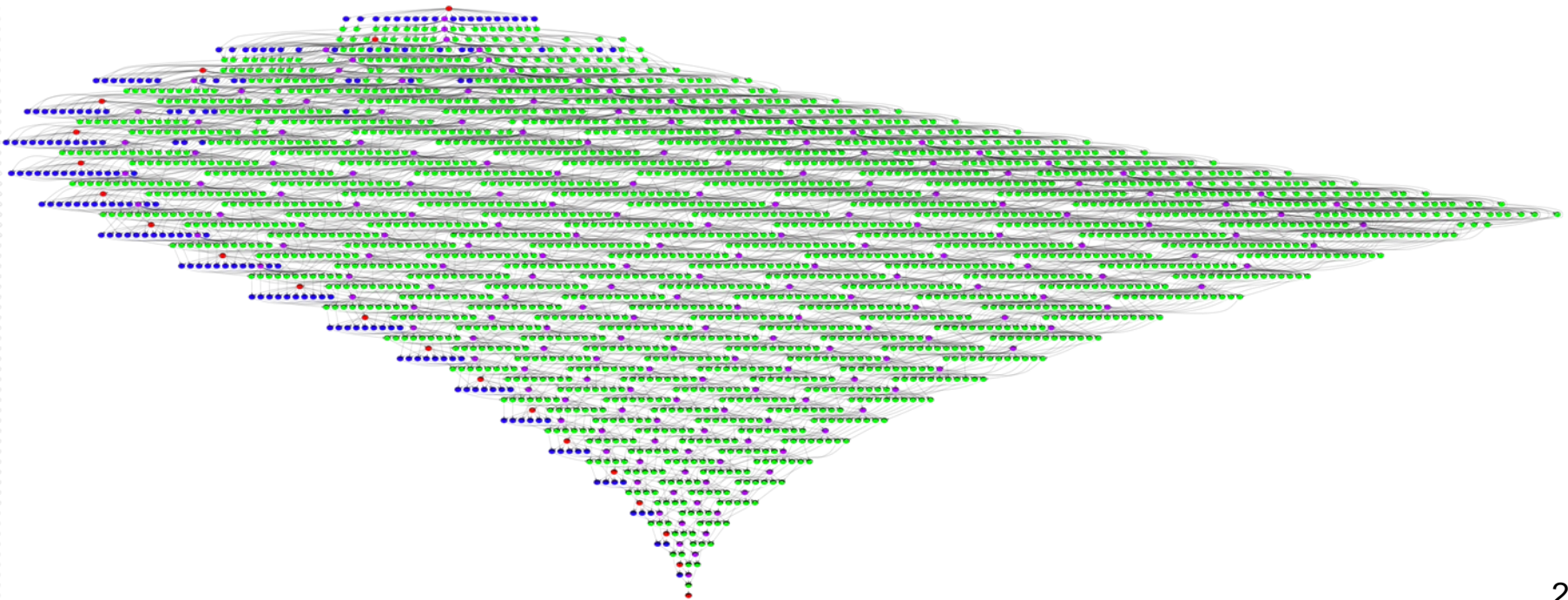
POTRF+TRTRI+LAUUM: $25(7t-3)$
 Cholesky Factorization alone: $3t-2$



Pipelined: $18(3t+6)$

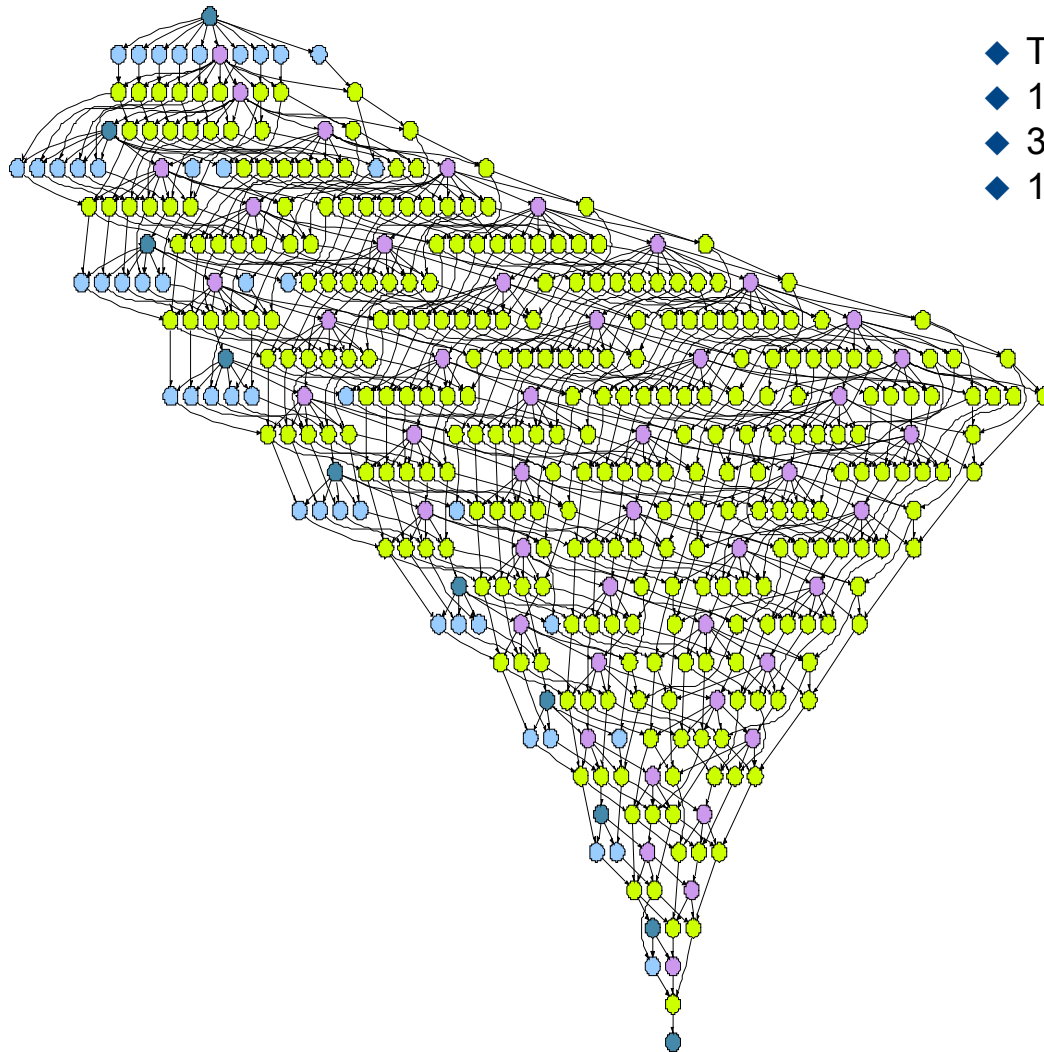
Big DAGs: No Global Critical Path

- DAGs get very big, very fast
 - So windows of active tasks are used; this means no global critical path
 - Matrix of $NB \times NB$ tiles; NB^3 operation
 - $NB=100$ gives 1 million tasks



PLASMA Scheduling

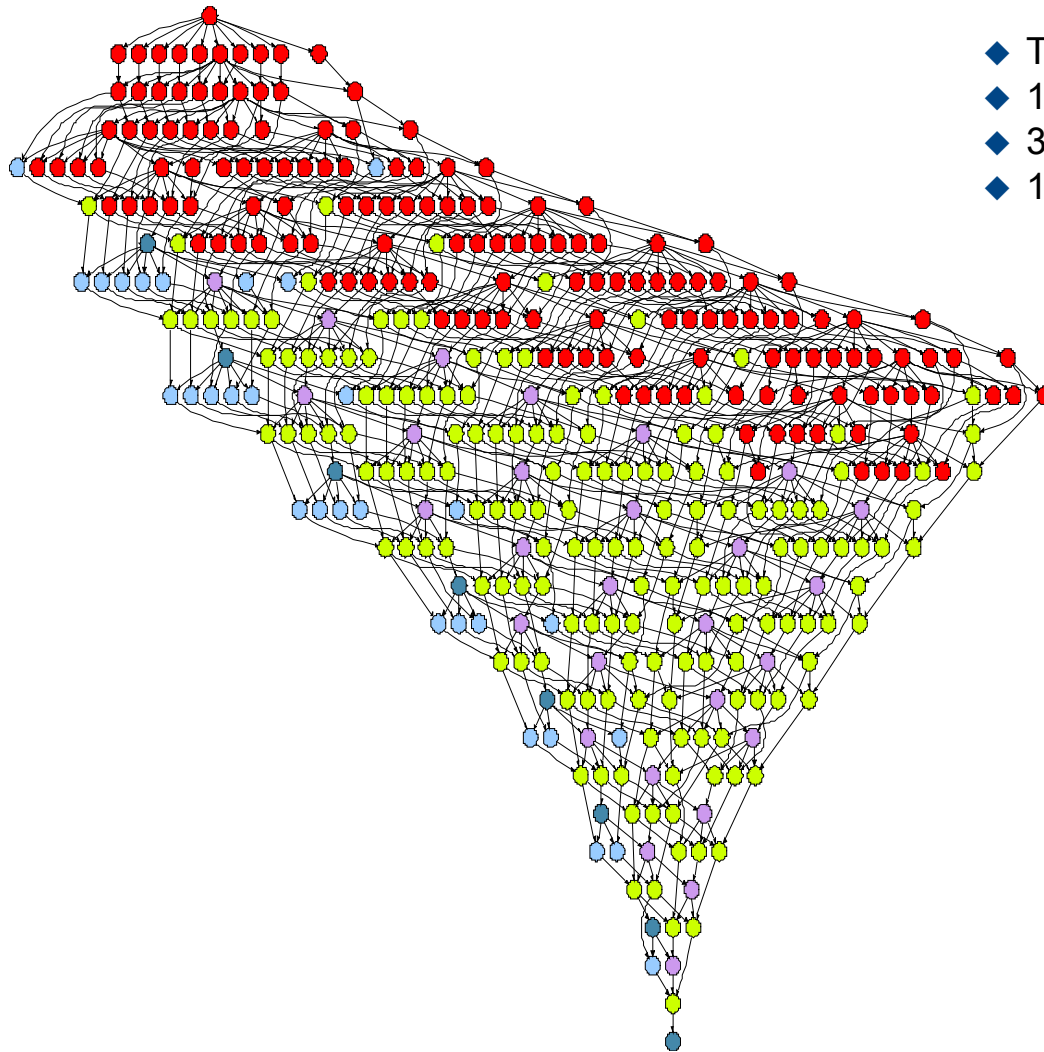
Dynamic Scheduling: Sliding Window



- ◆ Tile LU factorization
- ◆ 10 x 10 tiles
- ◆ 300 tasks
- ◆ 100 task window

PLASMA Scheduling

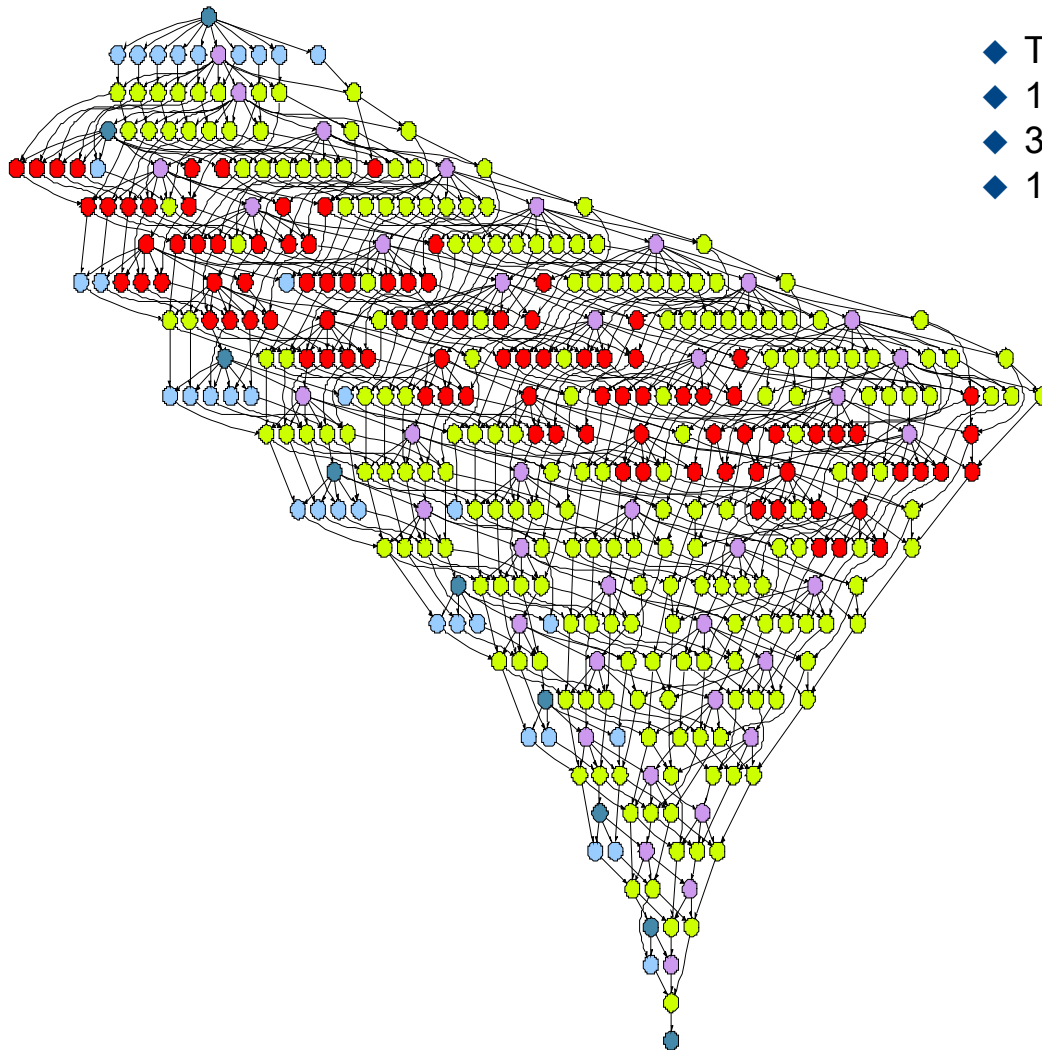
Dynamic Scheduling: Sliding Window



- ◆ Tile LU factorization
- ◆ 10 x 10 tiles
- ◆ 300 tasks
- ◆ 100 task window

PLASMA Scheduling

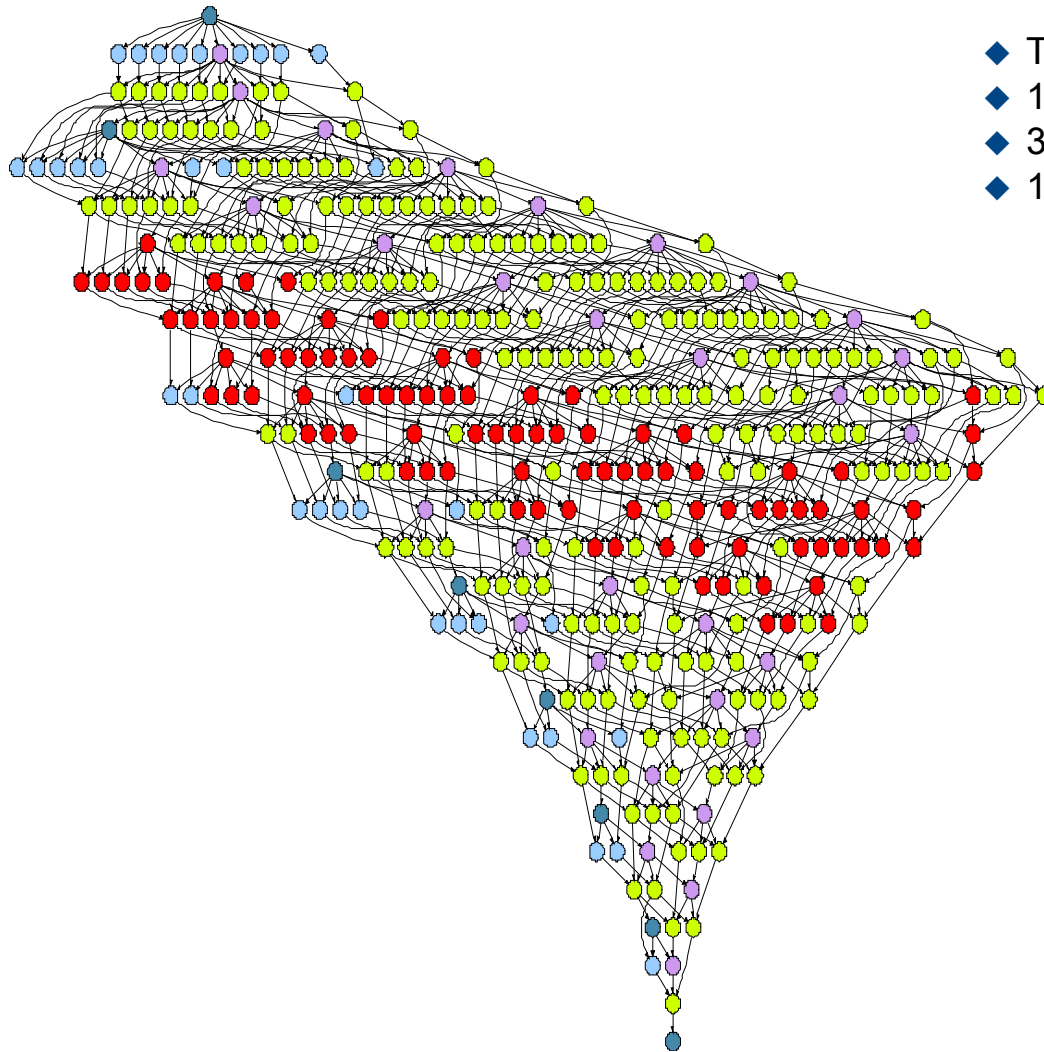
Dynamic Scheduling: Sliding Window



- ◆ Tile LU factorization
- ◆ 10 x 10 tiles
- ◆ 300 tasks
- ◆ 100 task window

PLASMA Scheduling

Dynamic Scheduling: Sliding Window



- ◆ Tile LU factorization
- ◆ 10 x 10 tiles
- ◆ 300 tasks
- ◆ 100 task window

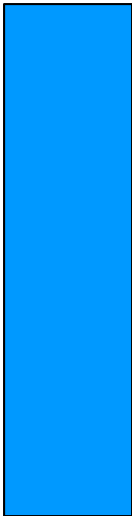
Communication Avoiding Algorithms

- **Goal: Algorithms that communicate as little as possible**
- **Jim Demmel and company have been working on algorithms that obtain a provable minimum communication.**
- **Direct methods (BLAS, LU, QR, SVD, other decompositions)**
 - **Communication lower bounds for *all* these problems**
 - **Algorithms that attain them (*all* dense linear algebra, some sparse)**
 - Mostly not in LAPACK or ScaLAPACK (yet)
- **Iterative methods - Krylov subspace methods for $Ax=b$, $Ax=\lambda x$**
 - **Communication lower bounds, and algorithms that attain them (depending on sparsity structure)**
 - Not in any libraries (yet)
- **For QR Factorization they can show:**

	Lower bound
# flops	$\Theta(mn^2)$
# words	$\Theta\left(\frac{mn^2}{\sqrt{W}}\right)$
# messages	$\Theta\left(\frac{mn^2}{W^{3/2}}\right)$

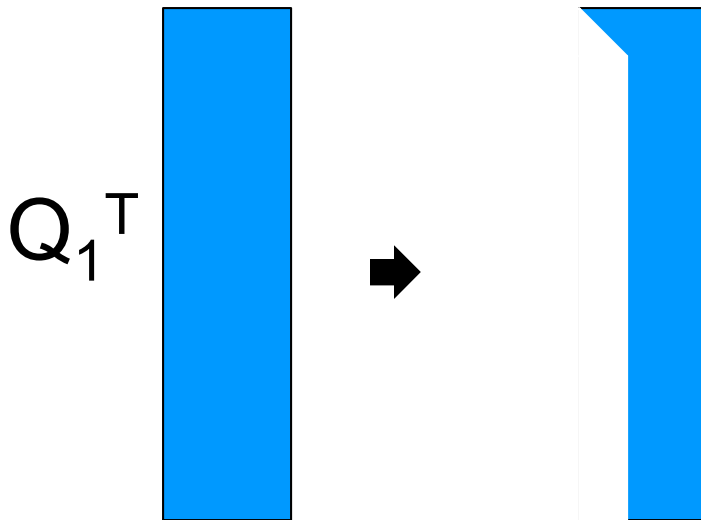
Standard QR Block Reduction

- We have a $m \times n$ matrix A we want to reduce to upper triangular form.



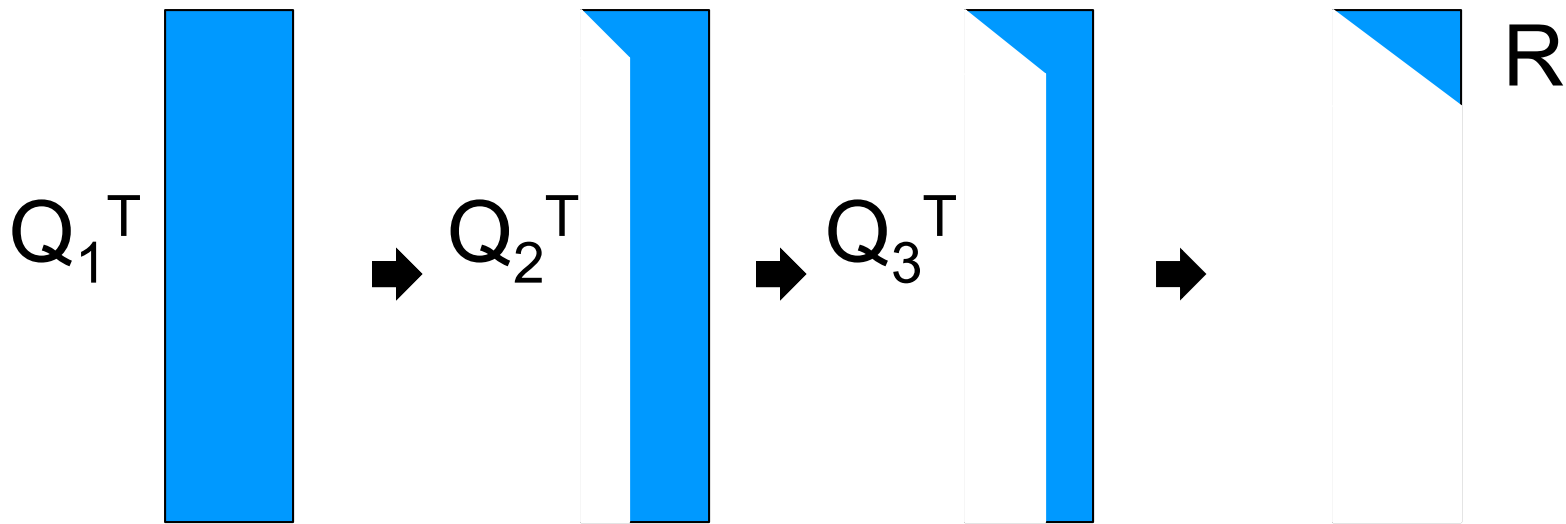
Standard QR Block Reduction

- We have a $m \times n$ matrix A we want to reduce to upper triangular form.



Standard QR Block Reduction

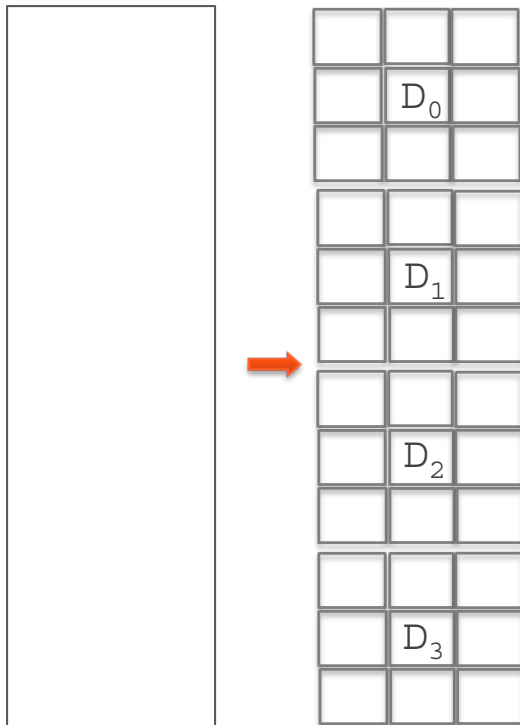
- We have a $m \times n$ matrix A we want to reduce to upper triangular form.



$$A = Q_1 Q_2 Q_3 R = QR$$

Communication Avoiding QR

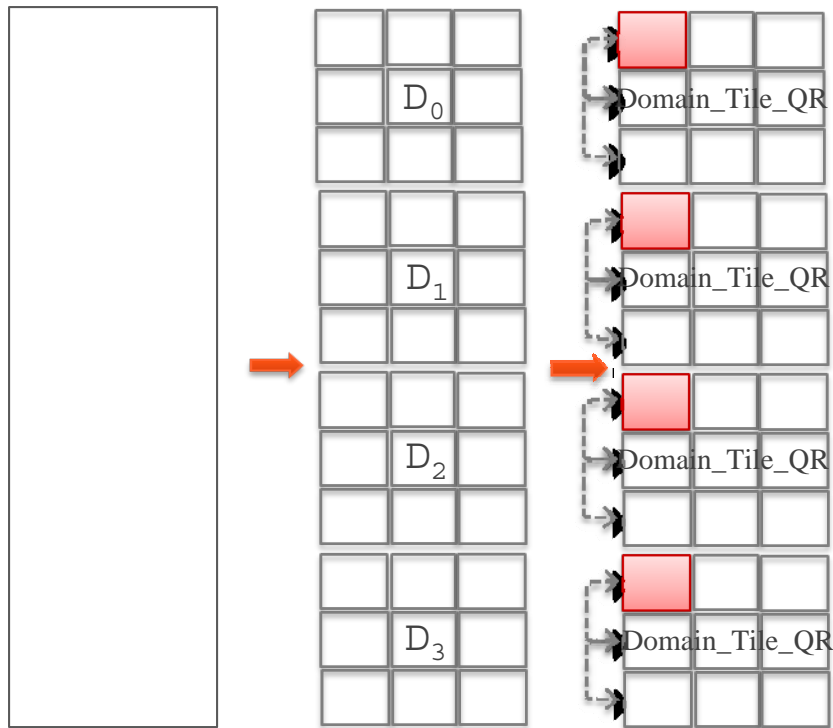
Example



A. Pothen and P. Raghavan. Distributed orthogonal factorization. In *The 3rd Conference on Hypercube Concurrent Computers and Applications, volume II, Applications*, pages 1610–1620, Pasadena, CA, Jan. 1988. ACM. Penn. State.

Communication Avoiding QR

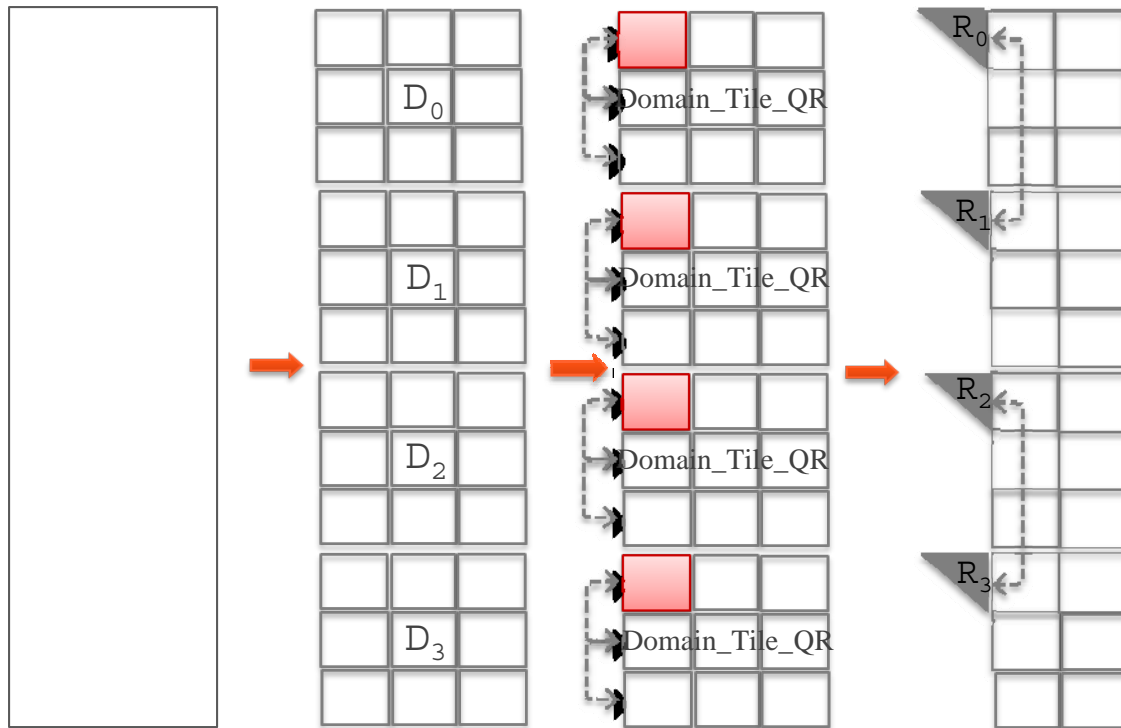
Example



A. Pothen and P. Raghavan. Distributed orthogonal factorization. In *The 3rd Conference on Hypercube Concurrent Computers and Applications, volume II, Applications*, pages 1610–1620, Pasadena, CA, Jan. 1988. ACM. Penn. State.

Communication Avoiding QR

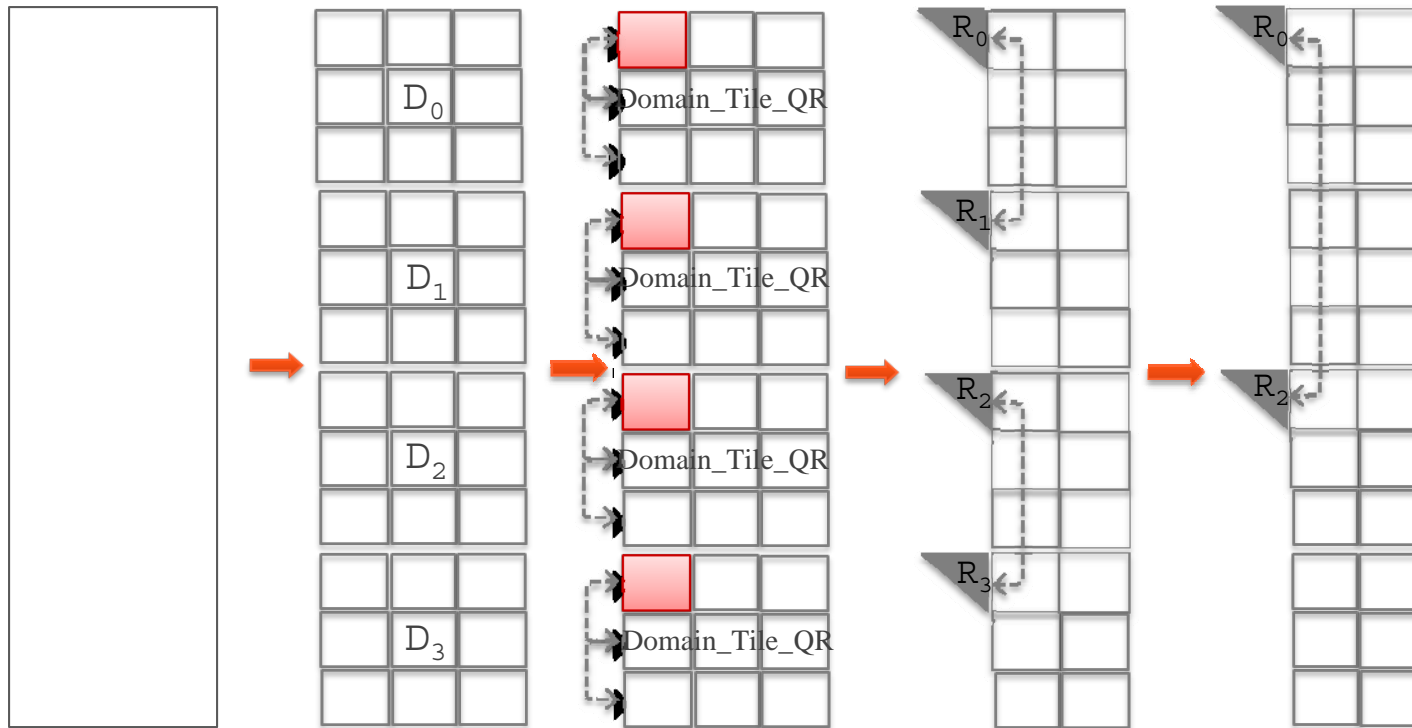
Example



A. Pothen and P. Raghavan. Distributed orthogonal factorization. In *The 3rd Conference on Hypercube Concurrent Computers and Applications, volume II, Applications*, pages 1610–1620, Pasadena, CA, Jan. 1988. ACM. Penn. State.

Communication Avoiding QR

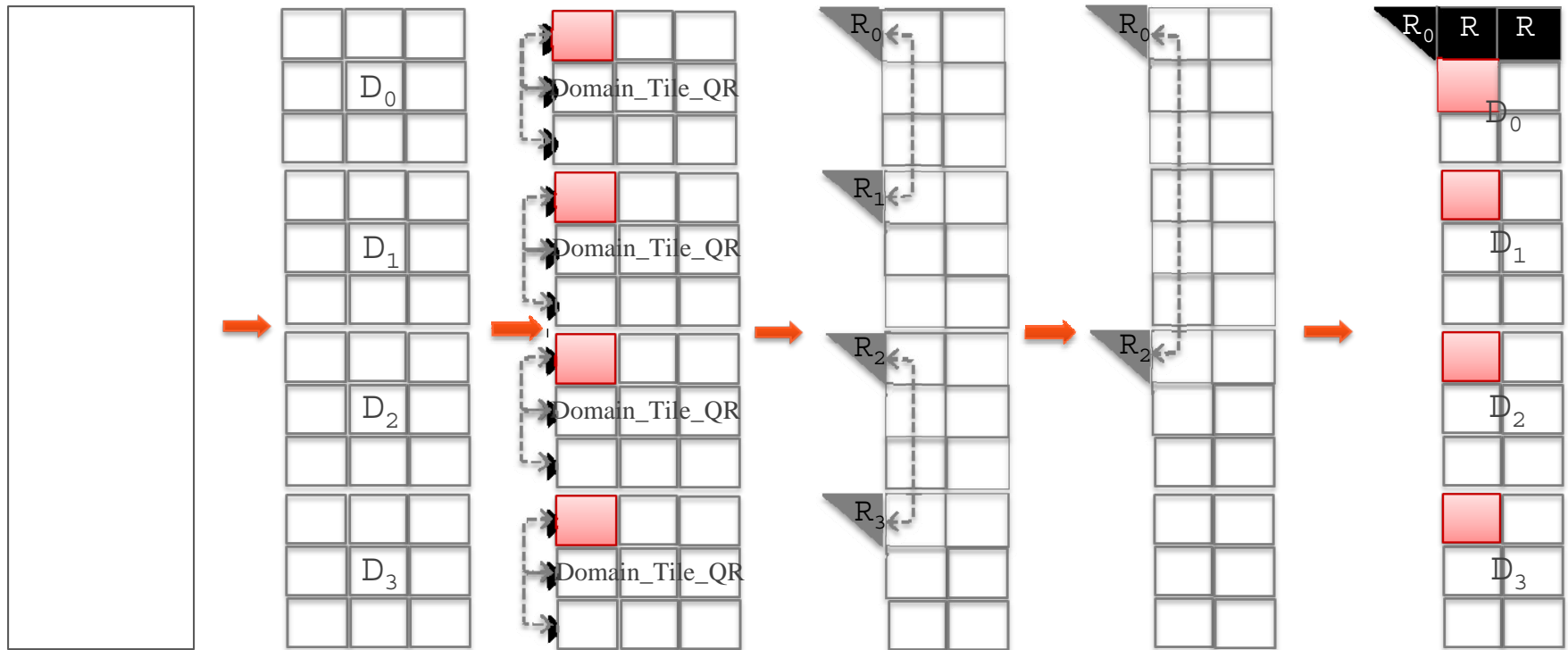
Example



A. Pothen and P. Raghavan. Distributed orthogonal factorization. In *The 3rd Conference on Hypercube Concurrent Computers and Applications, volume II, Applications*, pages 1610–1620, Pasadena, CA, Jan. 1988. ACM. Penn. State.

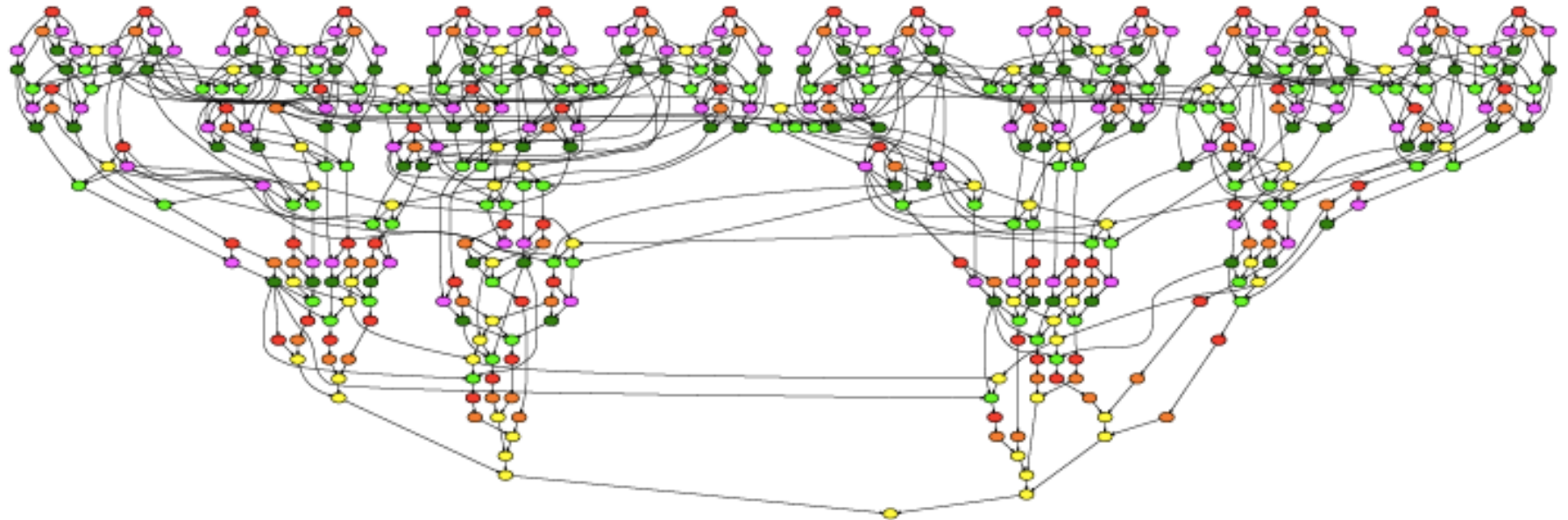
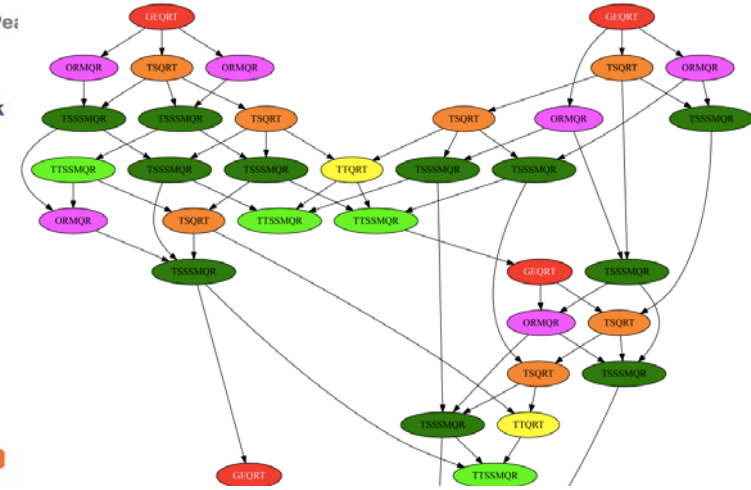
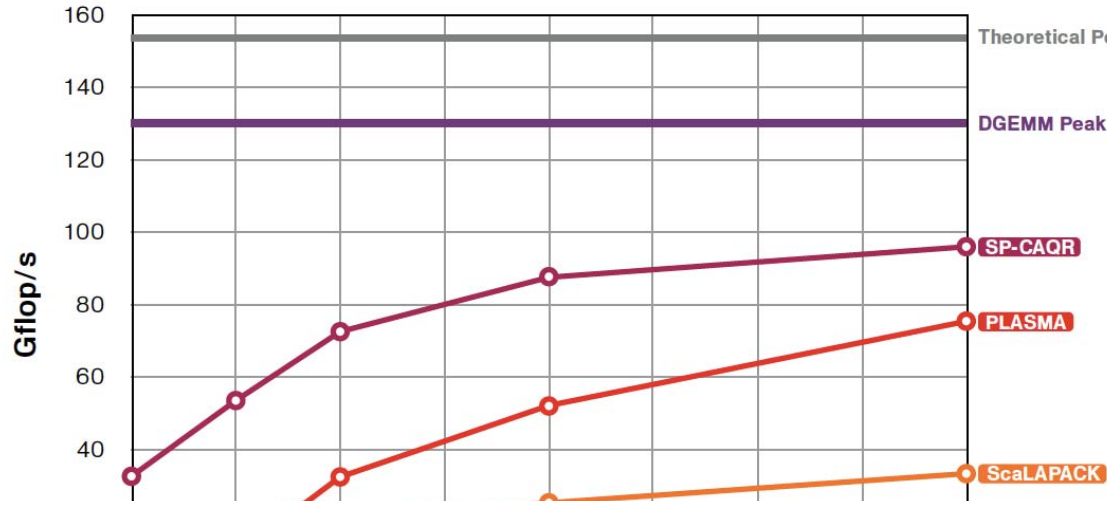
Communication Avoiding QR

Example



A. Pothen and P. Raghavan. Distributed orthogonal factorization. In *The 3rd Conference on Hypercube Concurrent Computers and Applications, volume II, Applications*, pages 1610–1620, Pasadena, CA, Jan. 1988. ACM. Penn. State.

Communication Reducing QR Factorization



Mixed Precision Methods

- **Mixed precision, use the lowest precision required to achieve a given accuracy outcome**
 - **Improves runtime, reduce power consumption, lower data movement**
 - **Reformulate to find correction to solution, rather than solution; Δx rather than x .**

Idea Goes Something Like This...

- **Exploit 32 bit floating point as much as possible.**
 - **Especially for the bulk of the computation**
- **Correct or update the solution with selective use of 64 bit floating point to provide a refined results**
- **Intuitively:**
 - **Compute a 32 bit result,**
 - **Calculate a correction to 32 bit result using selected higher precision and,**
 - **Perform the update of the 32 bit results with the correction using high precision.**

Mixed-Precision Iterative Refinement

- Iterative refinement for dense systems, $Ax = b$, can work this way.

$L U = \text{lu}(A)$	$O(n^3)$
$x = L \setminus (U \setminus b)$	$O(n^2)$
$r = b - Ax$	$O(n^2)$
WHILE $\ r \ $ not small enough	
$z = L \setminus (U \setminus r)$	$O(n^2)$
$x = x + z$	$O(n^1)$
$r = b - Ax$	$O(n^2)$
END	

- Wilkinson, Moler, Stewart, & Higham provide error bound for SP fl pt results when using DP fl pt.

Mixed-Precision Iterative Refinement

- Iterative refinement for dense systems, $Ax = b$, can work this way.

$L U = \text{lu}(A)$	SINGLE	$O(n^3)$
$x = L \setminus (U \setminus b)$	SINGLE	$O(n^2)$
$r = b - Ax$	DOUBLE	$O(n^2)$
WHILE $\ r \ $ not small enough		
$z = L \setminus (U \setminus r)$	SINGLE	$O(n^2)$
$x = x + z$	DOUBLE	$O(n^1)$
$r = b - Ax$	DOUBLE	$O(n^2)$
END		

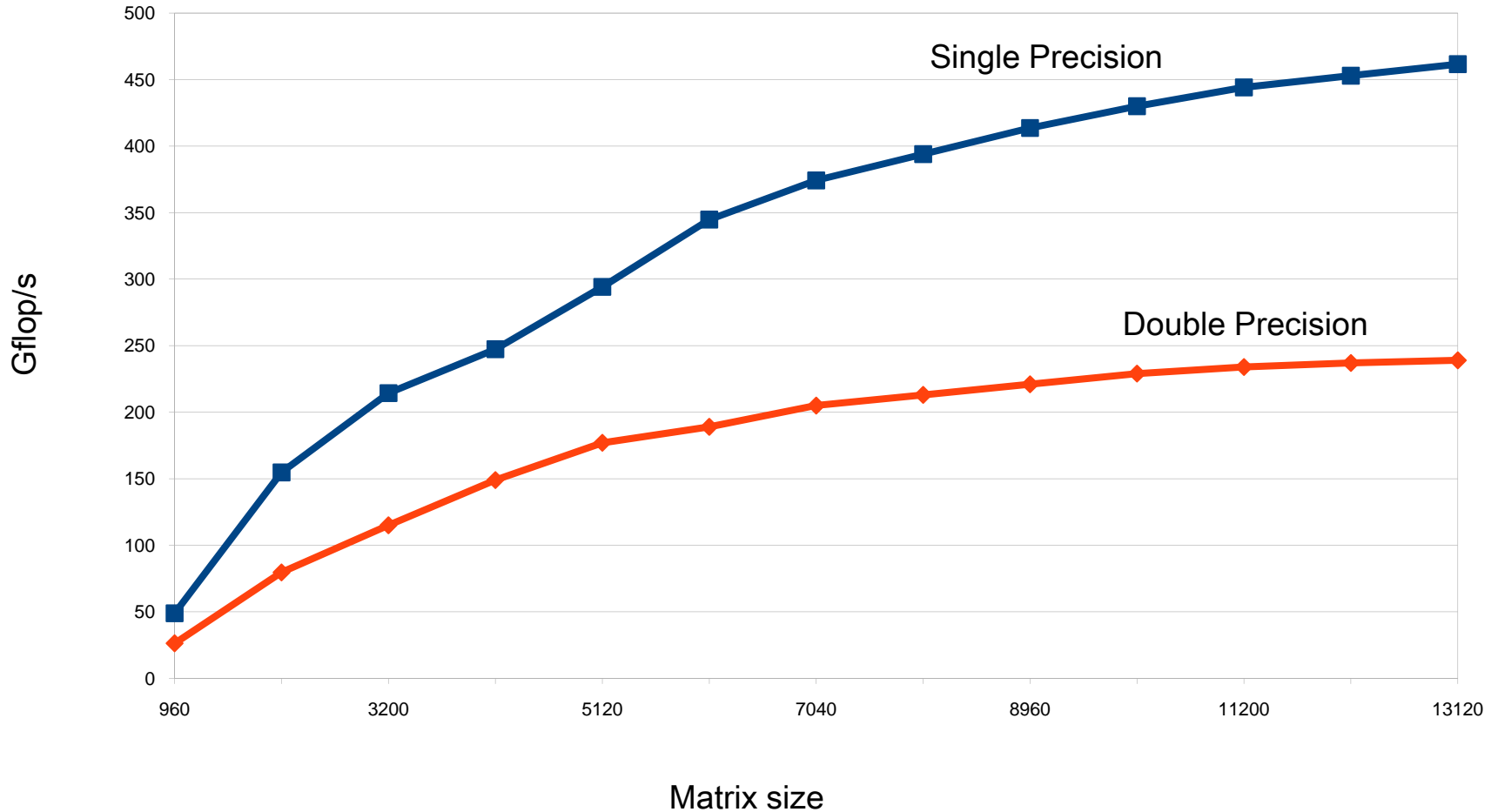
- Wilkinson, Moler, Stewart, & Higham provide error bound for SP fl pt results when using DP fl pt.
- It can be shown that using this approach we can compute the solution to 64-bit floating point precision.

- Requires extra storage, total is 1.5 times normal;
- $O(n^3)$ work is done in lower precision
- $O(n^2)$ work is done in high precision
- Problems if the matrix is ill-conditioned in sp; $O(10^8)$

$Ax = b$

FERMI

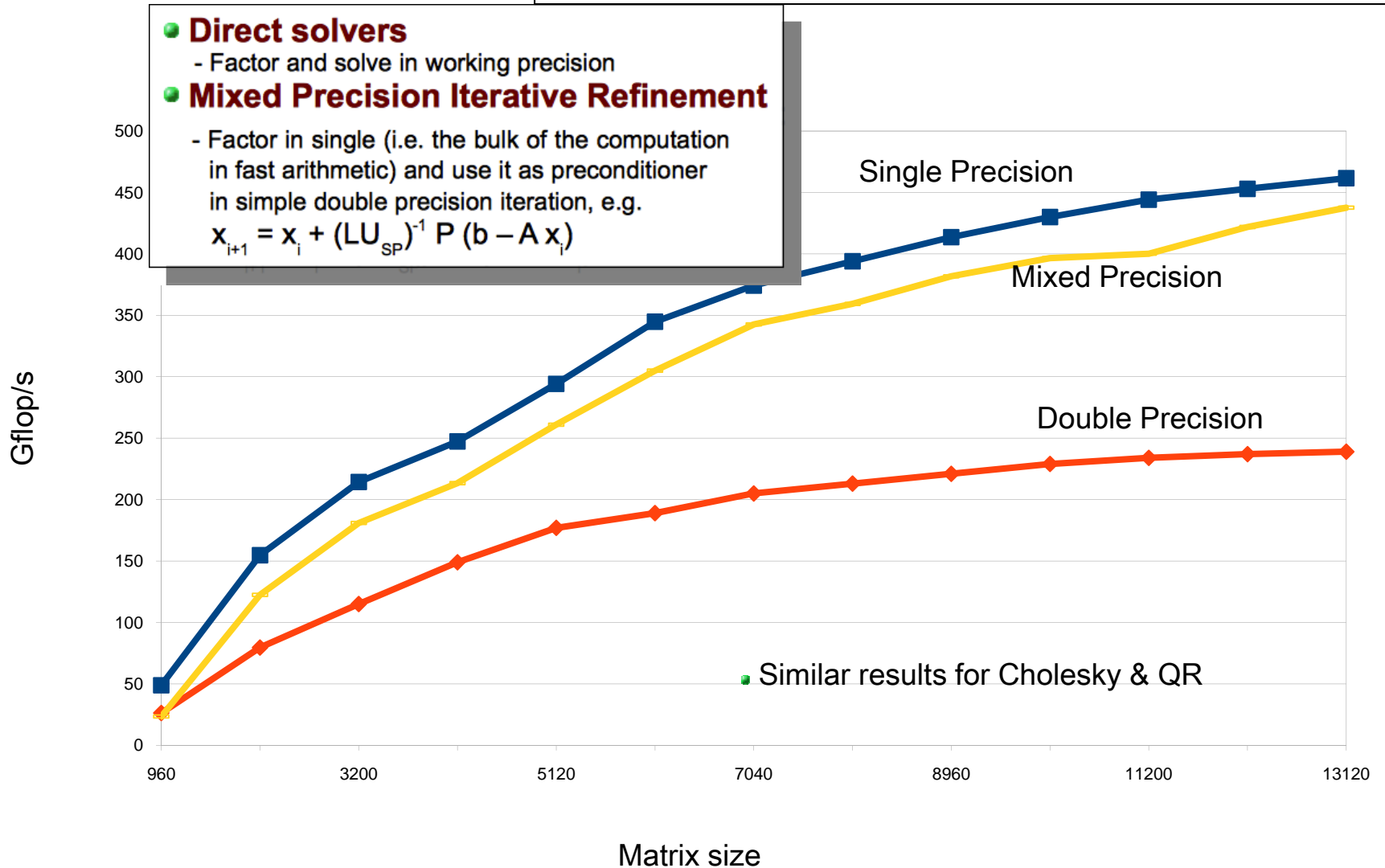
Tesla C2050: 448 CUDA cores @ 1.15GHz
SP/DP peak is 1030 / 515 GFlop/s



Ax = b

FERMI

Tesla C2050: 448 CUDA cores @ 1.15GHz
 SP/DP peak is 1030 / 515 GFlop/s



Power Profiles

Two dual-core 1.8 GHz AMD Opteron processors

Theoretical peak: 14.4 Gflops per node

DGEMM using 4 threads: 12.94 Gflops

PLASMA 2.3.1, GotoBLAS2

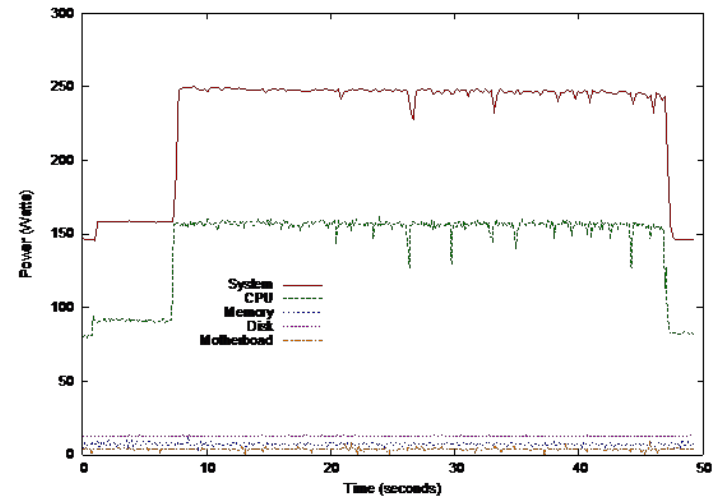
Experiments:

PLASMA LU solver in double precision

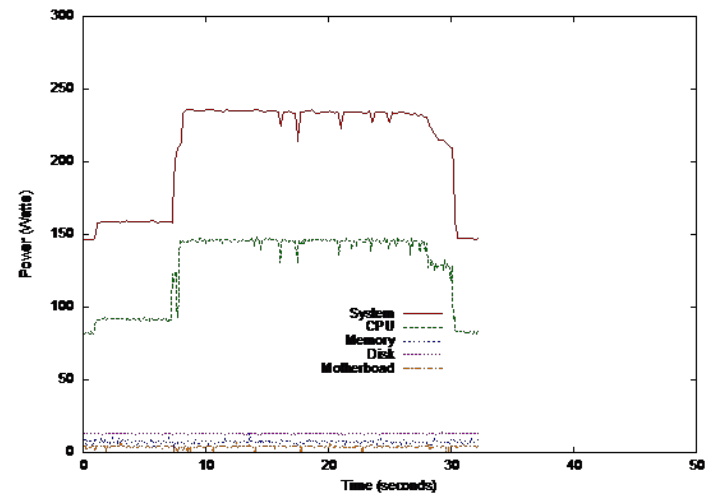
PLASMA LU solver in mixed precision

N = 8400, using 4 cores	PLASMA DP	PLASMA Mixed
Time to Solution (s)	39.5	22.8
GFLOPS	10.01	17.37
Accuracy $\frac{\ Ax - b\ }{(\ A\ \ X\ + \ b\)N\epsilon}$	2.0E-02	1.3E-01
Iterations		7
System Energy (KJ)	10852.8	6314.8

PLASMA DP



PLASMA Mixed Precision



Reproducibility

- For example $\sum x_i$ when done in parallel can't guarantee the order of operations.
- Lack of reproducibility due to floating point nonassociativity and algorithmic adaptivity (including autotuning) in efficient production mode
- Bit-level reproducibility may be unnecessarily expensive most of the time
- Force routine adoption of uncertainty quantification
 - **Given the many unresolvable uncertainties in program inputs, bound the error in the outputs in terms of errors in the inputs**

A Call to Action: Exascale is a Global Challenge



- Hardware has changed dramatically while software ecosystem has remained stagnant
- Community codes unprepared for sea change in architectures
- No global evaluation of key missing components
- The IESP was Formed in 2008
- Goal to engage international computer science community to address common software challenges for Exascale
- Focus on open source systems software that would enable multiple platforms
- Shared risk and investment
- Leverage international talent base

International Exascale Software Program

Improve the world's simulation and modeling capability by improving the coordination and development of the HPC software environment

Workshops:

Build an international plan for coordinating research for the next generation open source software for scientific high-performance computing

Example Organizational Structure: Incubation Period (today):



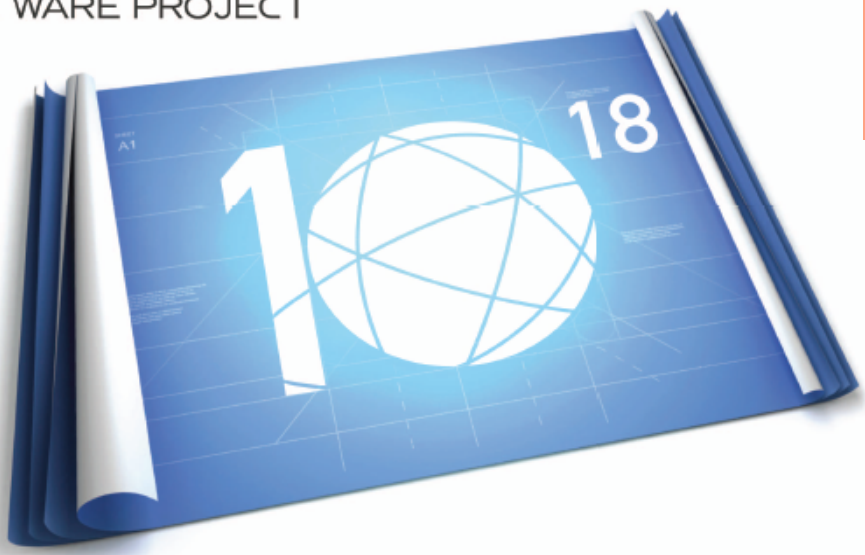
- **IESP provides coordination internationally, while regional groups have well managed R&D plans and milestones**

Conclusions

- For the last decade or more, the research investment strategy has been overwhelmingly biased in favor of hardware.
- This strategy needs to be rebalanced - barriers to progress are increasingly on the software side.
- Moreover, the return on investment is more favorable to software.
 - Hardware has a half-life measured in years, while software has a half-life measured in decades.
- High Performance Ecosystem out of balance
 - Hardware, OS, Compilers, Software, Algorithms, Applications
 - No Moore's Law for software, algorithms and applications

Published in the January 2011 issue of
The International Journal of High
Performance Computing Applications

48



- | | | | | |
|-----------------------|-----------------|------------------|-------------------|-------------------|
| Jack Dongarra | Alok Choudhary | Sanjay Kale | Matthias Mueller | Bob Sugar |
| Pete Beckman | Sudip Dosanjh | Richard Kenway | Wolfgang Nagel | Shinji Sumimoto |
| Terry Moore | Thom Dunning | David Keyes | Hiroshi Nakashima | William Tang |
| Patrick Aerts | Sandro Fiore | Bill Kramer | Michael E. Papka | John Taylor |
| Giovanni Aloisio | Al Geist | Jesus Labarta | Dan Reed | Rajeev Thakur |
| Jean-Claude Andre | Bill Gropp | Alain Lichnewsky | Mitsuhsa Sato | Anne Trefethen |
| David Barkai | Robert Harrison | Thomas Lippert | Ed Seidel | Mateo Valero |
| Jean-Yves Berthou | Mark Herald | Bob Lucas | John Shalf | Aad van der Steen |
| Taisuke Boku | Michael Heroux | Barney Maccabe | David Skinner | Jeffrey Vetter |
| Bertrand Braunschweig | Adolfy Hoisie | Satoshi Matsuoka | Marc Snir | Peg Williams |
| Franck Cappello | Koh Hotta | Paul Messina | Thomas Sterling | Robert Wisniewski |
| Barbara Chapman | Yutaka Ishikawa | Peter Michielse | Rick Stevens | Kathy Yelick |
| Xuebin Chi | Fred Johnson | Bernd Mohr | Fred Streit | |

“We can only see a short distance ahead, but we can see plenty there that needs to be done.”

▪ *Alan Turing (1912 – 1954)*

SPONSORS



• www.exascale.org