
MONTY PYTHON'S
And Now For Something
COMPLETELY DIFFERENT
DVD

Department of Business Administration
University of Illinois at Urbana-Champaign

Parallel Stochastic Programming: The DOD Airlift Allocation Problem

Akhil Langer, Ramprasad Venkataraman, Sanjay Kale, Udatta Palekar

University of Illinois at Urbana-Champaign

Steve Baker, Mark Surina

MITRE Corp.

9th Annual Charm++ Workshop 2011

April 19, 2011

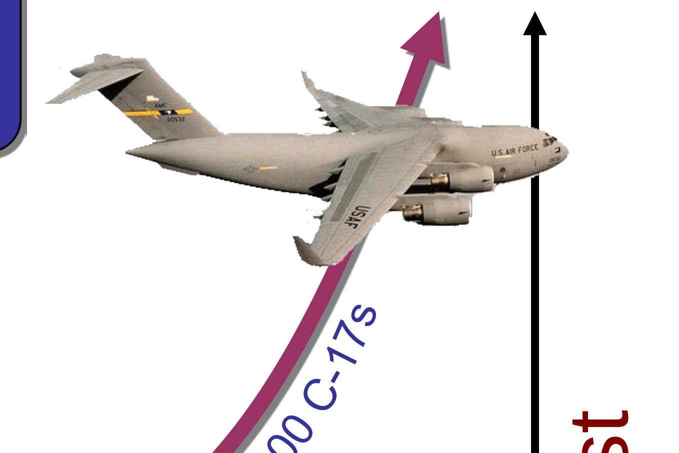
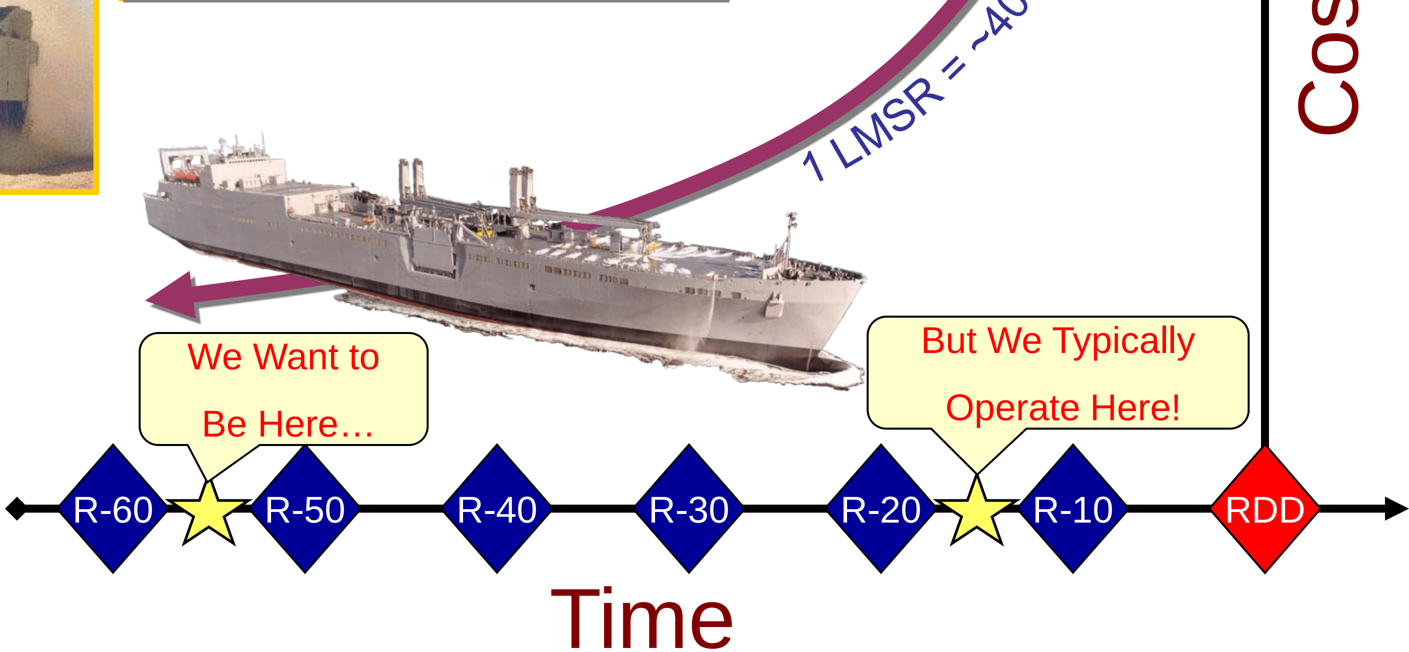
On Any Given Day……..

- USTRANSCOM must handle
- 100 railcar shipments
- 35 ships loading, offloading, or underway
- 1,000 truck shipments
- 480 airlift sorties
  - 310 Military
  - 170 Commercial
- 70 operational air refueling sorties
- 7 air evacuation sorties
- Aircraft takeoff or landing every 90 seconds

Mobility Tradeoffs

Tank tracks
(125 containers)
Air: $17.5M
Sea: $364K

Concrete (16,954 TONS)
Air: $129M   Sea: $5.5M

Time?
3-4 Weeks (ship)
vs.
2-3 Days (aircraft)

Constrained Resources… Premium on Right Asset, Right Mission!

Air Mobility Command

HQ: Scott AFB, IL

MISSION:

"Provide airlift, air refueling, special air mission, and aeromedical evacuation for U.S. forces."

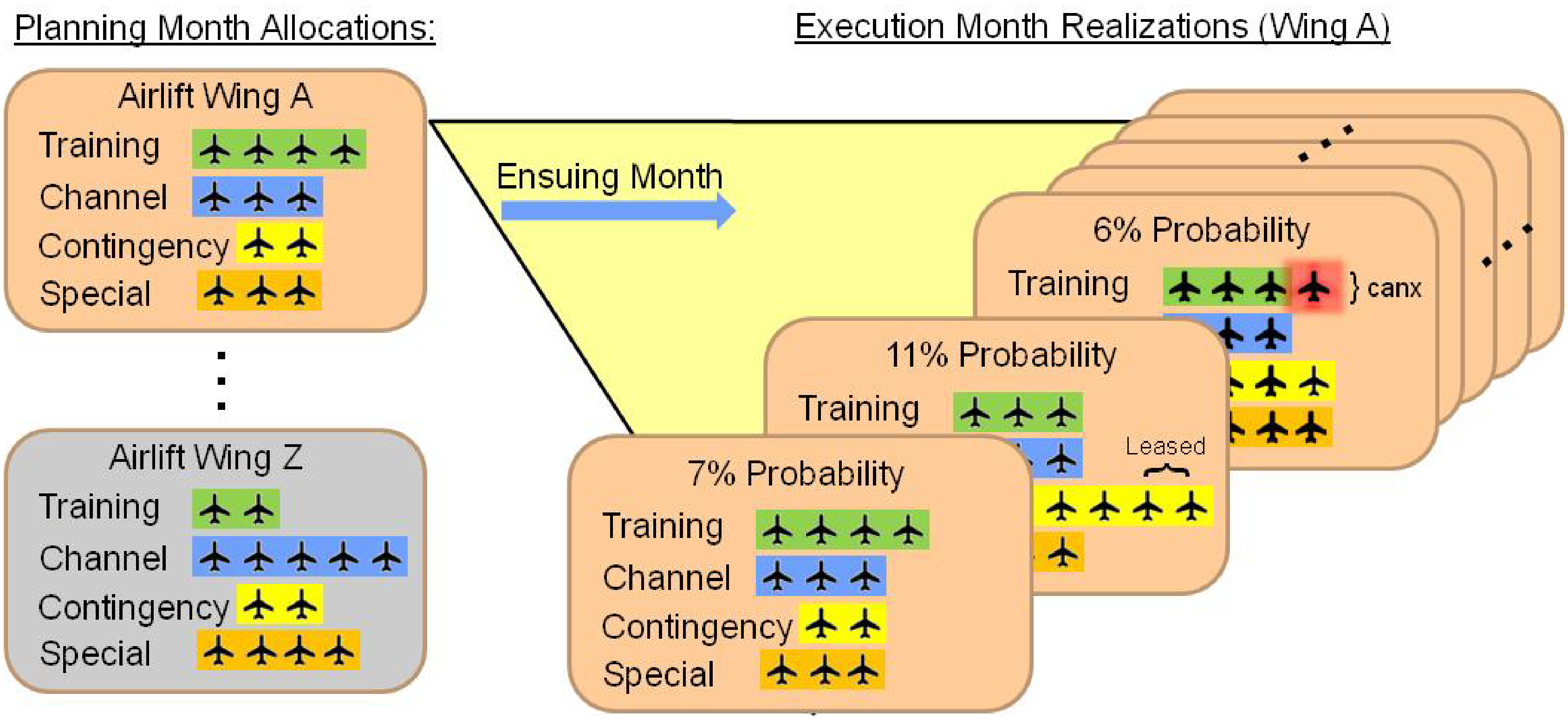
- Worldwide Airlift
- Worldwide Air Refueling
- Aeromedical Evacuation
- Presidential & DV Support
- Civil Reserve Air Fleet (CRAF)

Background

- Management of the DoD air transportation system lacks the optimal strategies for decision support that the private sector relies heavily upon
  - DoD manages the world's largest airline with uniquely diverse missions
  - Even in peacetime, mission requirements are subject to enormous uncertainty

- **The Tanker Airlift Control Center (TACC) must reconcile this diverse uncertainty when predicting monthly aircraft utilization**

Problem Context



- **Tanker Airlift Control Center (TACC) allocations to wings incorporate a “best guess” of next month’s requirements**
  - **Myriad possible outcomes confound decision support, e.g., aircraft breakdowns, weather, natural disaster, conflict**

Modeling Approach

Minimize:

1. The costs of allocating military and long-term leased aircraft to mission categories (Stage 1)

+

2. The ***expected*** costs of short-term aircraft leasing, aircraft operating and late and non-delivered cargo (Channel, Contingency) and missed missions (SAAM, Training) (Stage 2)

- Combine stochastic programming with parallel computing to model allocation of aircraft to airlift mission types during a periodic planning cycle
  - Stochastic programming addresses the highly probabilistic nature of military airlift: a traditional downfall of optimization in this environment
  - Parallel computing facilitates reconciliation of myriad possible outcomes in a timely manner

Solving the resulting Stochastic Program (Bender's Method)

$$\min_{y \epsilon R_+^n : Ay=b} \{c^t y + E_\omega[stage\ 2\ costs]\}$$

$$where\ \ E_\omega[stage\ 2\ costs] = E_\omega\left[\min_{x \epsilon R_+^p}\{q^t y : Wy = h(\omega) - Tx\}\right] = \sum_\omega p_\omega\, \vartheta_\omega$$

Linear Program

$$\vartheta_\omega = \left(\min_{x \epsilon R_+^p}\{q^t y : Wy = h(\omega) - Tx\}\right) = \left(\max_{x \epsilon R_+^m}\{(h(\omega) - Tx)^t v : W^t v = q\}\right)$$

$$= \max_{v^i \in \{v^1, v^2, \cdots, v^k\}} (h(\omega) - Tx)^t v^i$$

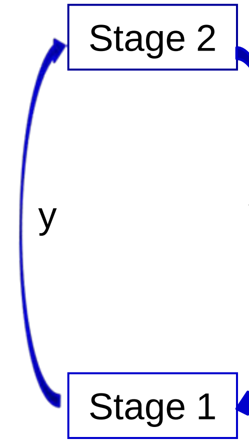
Linear Program

$$\min_{y \epsilon R_+^n : Ay=b} \{c^t y + E_\omega[stage\ 2\ costs]\}$$

$$= \min_{y \epsilon R_+^n} \left\{c^t y + \sum_\omega p_\omega\, \vartheta_\omega\right\}$$

$$s.t. \quad Ay = b$$

$$\vartheta_\omega \geq \max_{v^i \in \{v^1, v^2, \ldots\}} (h(\omega) - Tx)^t v^i \qquad \forall \omega$$



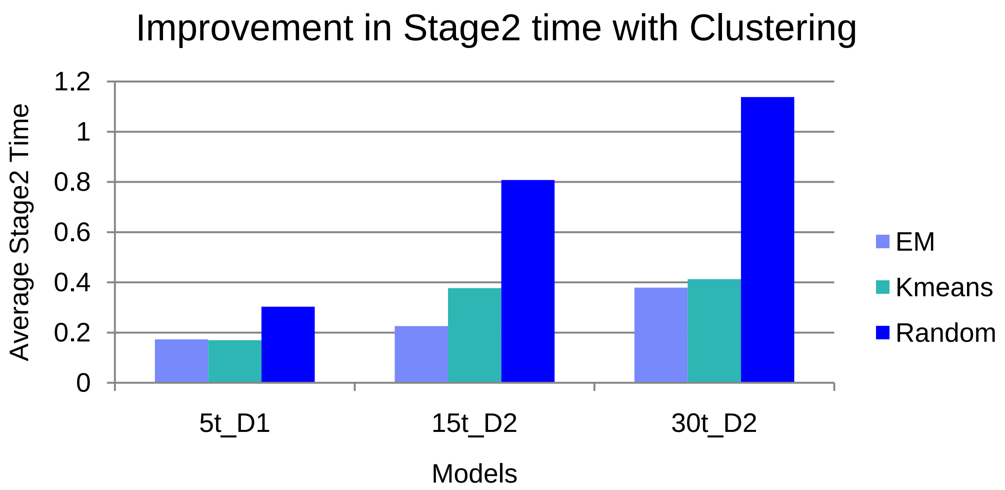
Lower and Upper bounds can be calculated to detect convergence

Parallel Implementation in CHARM++

- With a large number of stage 2 scenarios
  - Obvious gross parallelism – Solve scenarios on multiple cores
- Some things to note:
  - Cannot trivially break down individual stage 2 problems
    - LPs solved using Simplex Method
  - Each LP is large and can take significant amount of solution time
  - Scenario solve times can be highly variable
  - Messages sent will be very large if each scenario must be transmitted to its requesting processor
    - Dedicated processors for solving stage 1 and stage 2 problems
    - Each processor has a copy of the model
    - Need only pass the "RHS" to set up the correct scenario

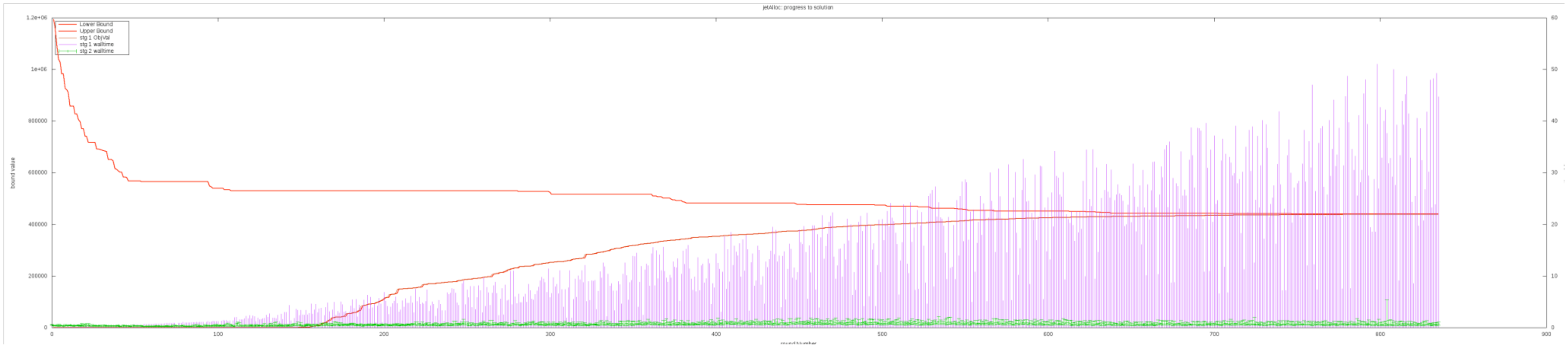
Dependence between Stage 2 scenarios

- Each scenario can be solved starting from optimal dual basis of last scenario solved
  - Solve times depend on order in which scenarios are solved (not known a priori)

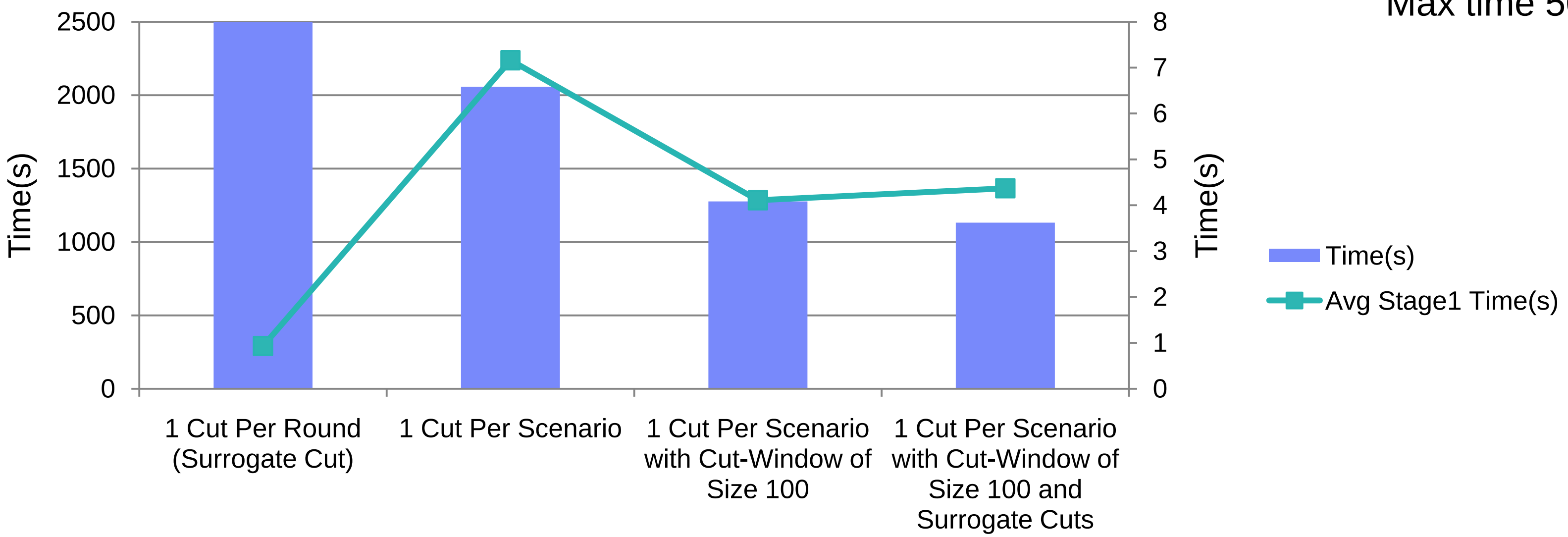


Solution – Clustering

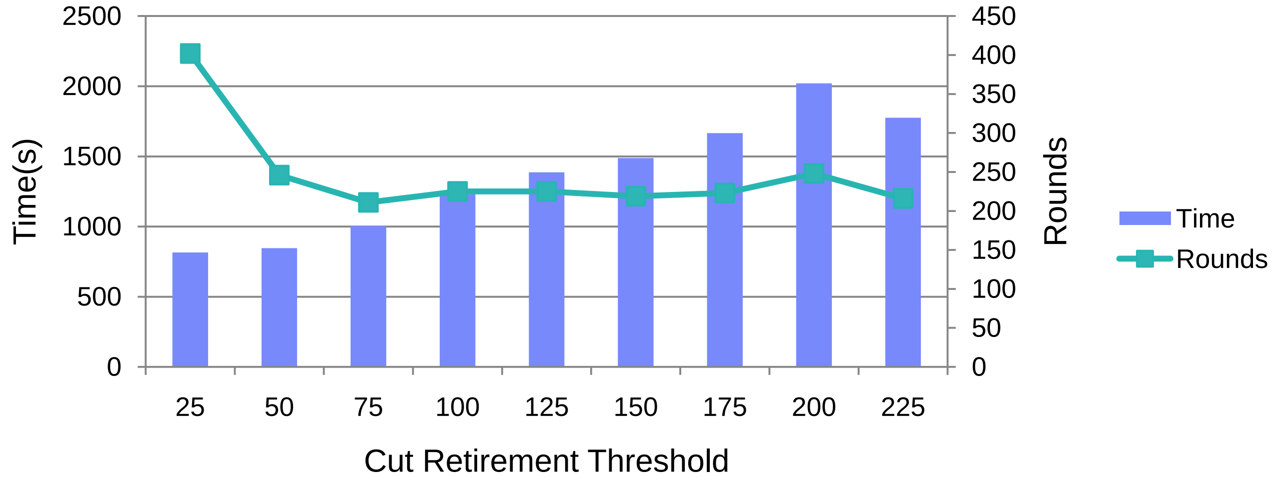
Growth of Stage 1 Solve times

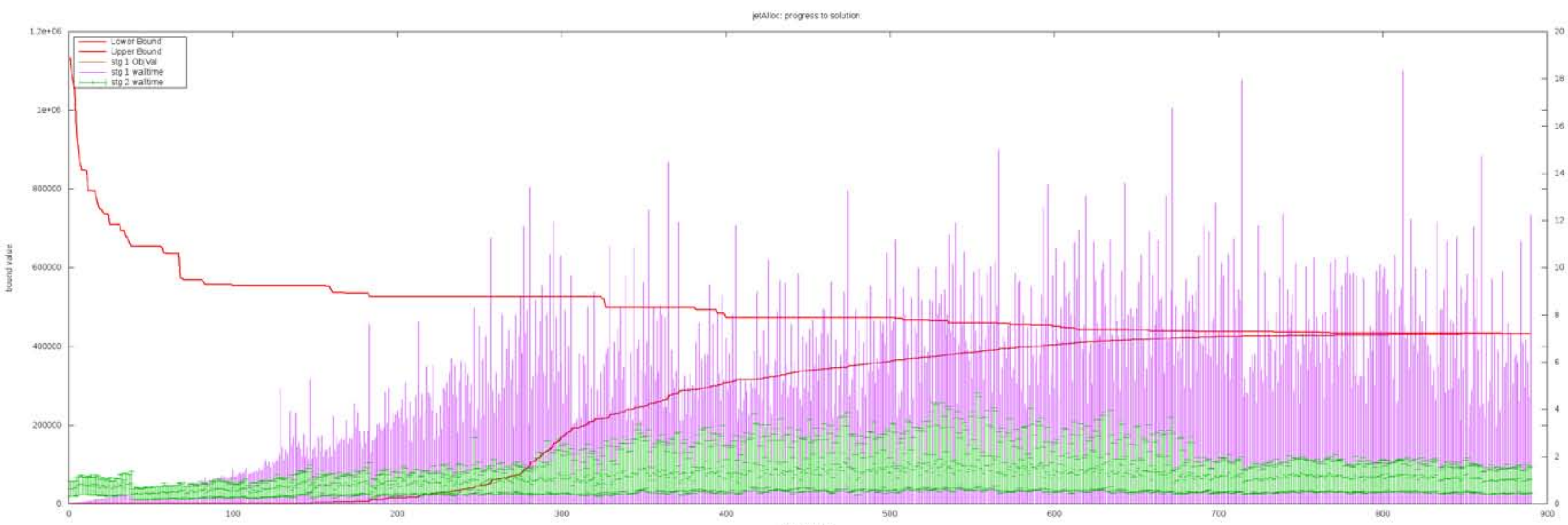


Max time 50+ secs



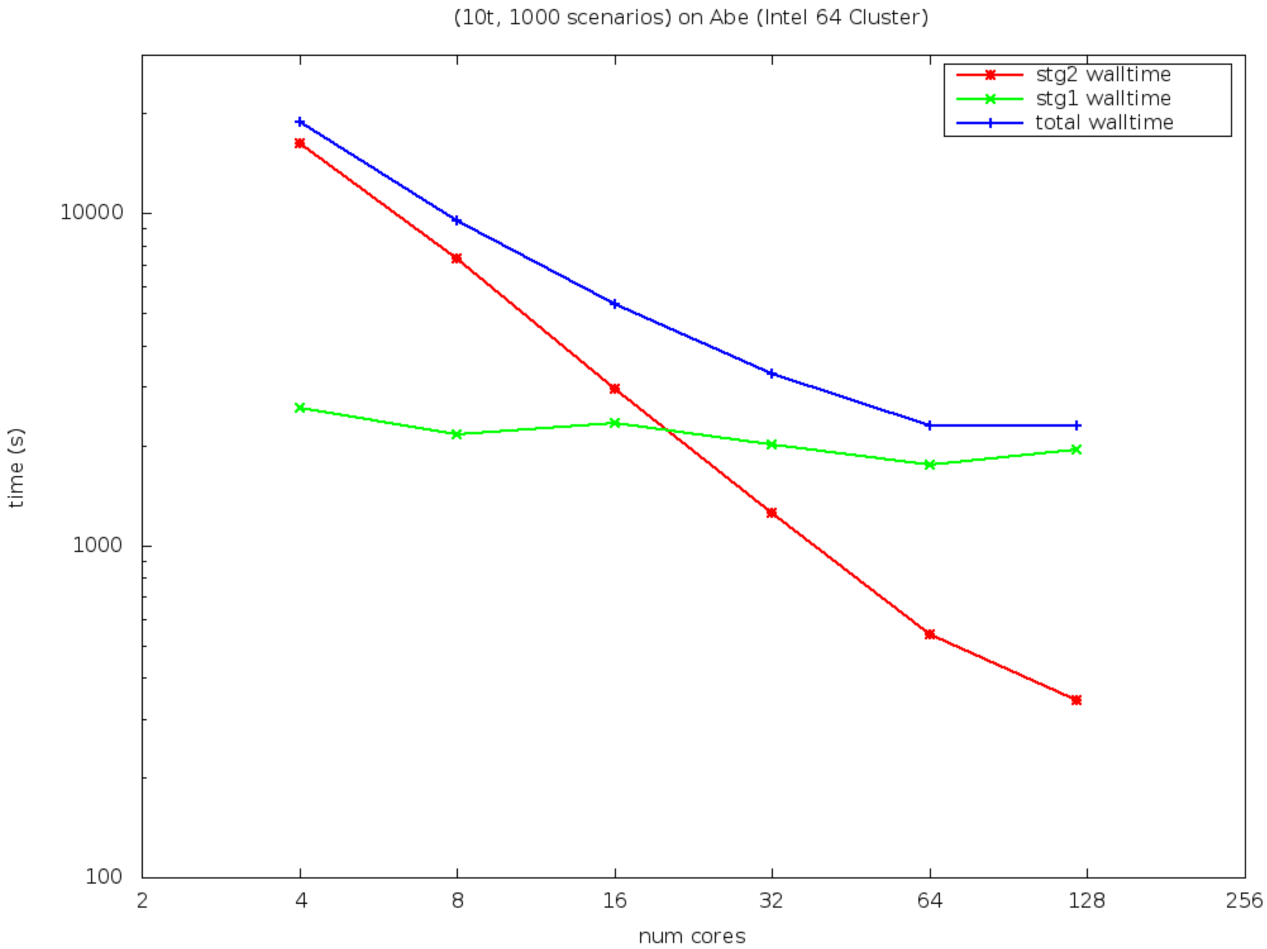
Cut retirement scheme





Max is 18 versus 50 without cut retirement

Scalability



Next Step: Mixed Integer Stochastic Program

- Allocations – stage 1 ”y variables” must be integral
  - Two approaches
    - Solve Stage 1 problem as an integer program
      - Cumbersome – must solve increasingly larger integer programs at each round
      - Inefficient – Nothing from prior rounds can be kept for succeeding rounds
  - Branch and Bound -Solve Stochastic LP at each node of the Branch and Bound tree
      - Benders cuts generated at any node of the tree are valid at all nodes of the tree
      - Each node inherits the enhanced LP of its parent node and can add more cuts as required
      - Can progressively tighten convergence tolerance as we go deeper down the tree where we are more likely to prune.
      - Since Stage 1 becomes an increasing bottleneck, we can buffer stage 2 processors by creating sufficient BnB nodes to keep stage 2 processors occupied
      - Rich parallel structure allows (will require) more efficient prioritization and scheduling schemes
- What about integral stage 2 variables?
  - Each scenario becomes an integer program!
  - Every terminated node of the “y variable tree” is a root for an integer program with M*S integer variables!
  - May not be practical to solve optimally.

Backup Slides

Algebraic Formulation

Stage One Formulation

$$
\begin{aligned}
\min \quad & \sum_j TP(R_j^1 \sum_l \tilde{y}_{j,l}) + \sum_{j \in J_{Mil},l,m,t} Ch_j y_{j,l,m,t} + \sum_s \pi_s \Theta_s(\mathbf{y},\mathbf{I}) \\
\text{s.t.} \quad & \\
\textit{Training Allocation}: \quad & \sum_{f,p:l_p=l,j_p=j} I_{f,p,t} \leq y_{j,l,4} \quad \forall t \\
\textit{Feasible Allocation1}: \quad & \sum_m y_{j,l,m,t} \leq Y_{j,l} \quad \forall j \in J_{Mil}, l, t \\
\textit{Feasible Allocation2}: \quad & \sum_m y_{j,l,m,t} - \tilde{y}_{j,l} \leq Y_{j,l} \quad \forall j \in J_{Civ}, l, t \\
\textit{LeasedAircraft} \quad & \tilde{y}_{j,l} \leq \tilde{Y}_{j,l} \quad \forall j \in J_{Civ}, l
\end{aligned}
$$

$$
\begin{aligned}
\textit{Stage 2 Cuts}: \quad \theta_s \geq \; & Opt_s(\mathbf{y}^*, \mathbf{I}^*) + \sum_{j \in G_1,l,m,t} (A_{j,t}(\omega) v^s_{7,j,l,m,t} + \mu_j v^s_{9,j,l,m})(y_{j,l,m,t} - y^*_{j,l,m,t}) \\
& + \sum_{j \in G_1,l,m} (A_{j,t}(\omega)(v^s_{7,j,l,t} + \mu_j v^s_{9,j,l}) \sum_{\{p:p \in AV_{t'}, j_p=j, l_p=l,t\}} (I_{\hat{f},p,t} - I^*_{\hat{f},p,t}))
\end{aligned}
$$

Algebraic Formulation (cont.)

Stage Two Formulation

$$Opt_s(\mathbf{y},\mathbf{I}) = \min \sum_{j,l,m,t} R_j^2 \hat{y}_{j,l,m,t}^2 + \sum_{i,k,t} P_{k,1}^1 u_{i,k,1,t}^1 + \sum_{i,k,t=rdd(i)}^{rdd(i)+t_{i,2}} P_{k,2}^1 u_{i,k,2,t}^1 + \sum_{i,k,m,t} P_{k,m}^2 u_{i,k,m,t}^2 + \sum_{r,j,m,t} E_{r,j} x_{r,j,t}^m + \ldots$$

s.t.

Channel

Demand1 :

$$-(u_{i,k,1,t-1}^1 + u_{i,k,1,t-1}^2) + \sum_{r\in S_{i,j}} \sum_{j} z_{i,j,k,r,t}^1 + u_{i,k,1,t}^1 + u_{i,k,1,t}^2 = D_{i,k,t}^1(\omega) \quad \forall i \notin \mathcal{TS}, k, t$$

Demand2 :

$$-(u_{i,k,1,t-1}^1 + u_{i,k,1,t-1}^2) + \sum_{r\in S_{i,j}} \sum_{j\in J_{Mil}} z_{i,j,k,r,t}^1 + \sum_{r\in S1_i} \sum_{j\in J_{Civ}} z_{i,j,k,r,t}^1 + u_{i,k,1,t}^1 + u_{i,k,1,t}^2 = D_{i,k,t}^1(\omega) \quad \forall i \in \mathcal{TS}, k, t$$

Transshipment :

$$\sum_{r\in S1_i, j\in J_{Civ}} z_{i,j,k,r,(t-\Delta_r^{i,j})}^1 - \sum_{r\in S2_i, j\in J_{Mil}} z_{i,j,k,r,t}^1 = 0 \quad \forall i \in \mathcal{TS}, k, t$$

Aggregate capacity :

$$\sum_{k\in K_j} z_{i,j,k,r,(t+\Delta_r^{i,j})}^1 - C_{r,i,j}^1 x_{r,j,(t-\Delta_r^{i,j})}^1 - \hat{C}_{i,j,t}(\omega) x_{r,j,(t-\Delta_r^{i,j})}^3 \le 0 \quad \forall i, j, r, t$$

Specific capacity :

$$z_{i,j,k,r,(t+\Delta_r^{i,j})}^1 - C_{r,i,j}^k x_{r,j,t}^1 \le 0 \quad \forall i, j, k, r, t$$

Price Break :

$$0 \le u_{i,k,1,t-1}^1 \le H_{i,k,t}^1 \quad \forall i, k, m, t$$

Algebraic Formulation (cont.)

Contingency

$$\textit{Demand1}: \quad -(u^1_{i,k,2,t-1} + u^2_{i,k,2,t-1}) + \sum_{r \in S_{i,j}} \sum_{j} z^2_{i,j,k,r,t} + u^1_{i,k,2,t} + u^2_{i,k,2,t} = D^2_{i,k,t}(\omega) \quad \forall i \notin \mathcal{TS}, k, t$$

$$\textit{Demand2}: \quad -(u^1_{i,k,2,t-1} + u^2_{i,k,2,t-1}) + \sum_{r \in S_{i,j}} \sum_{j \in J_{Mil}} z^2_{i,j,k,r,t} + \sum_{r \in S1_i} \sum_{j \in J_{Civ}} z^2_{i,j,k,r,t} + u^1_{i,k,2,t} + u^2_{i,k,2,t} = D^2_{i,k,t}(\omega) \quad \forall i \in \mathcal{TS}, k, t$$

$$\textit{Transshipment}: \quad \sum_{r \in S1_i, j \in J_{Civ}} z^2_{i,j,k,r,(t-\Delta^{i,j}_r)} - \sum_{r \in S2_i, j \in J_{Mil}} z^2_{i,j,k,r,t} = 0 \quad \forall i \in \mathcal{TS}, k, t$$

$$\textit{Aggregate capacity}: \quad \sum_{k \in K_j} \sum_{i:OD(i)=odi} z^2_{i,j,k,r,(t+\Delta^{i,j}_r)} - C^2_{r,odi,j} x^2_{r,j,t} \le 0 \quad \forall odi, j, r, t$$

$$\textit{Specific capacity}: \quad \sum_{i:OD(i)=odi} z^2_{i,j,k,r,(t+\Delta^{i,j}_r)} - C^k_{r,i,j} x^2_{r,j,t} \le 0 \quad \forall odi, j, r, t$$

$$\textit{Price Break}: \quad 0 \le u^1_{i,k,2,t-1} \le H^2_{i,k,t} \quad \forall i, k, m, t$$

SAAM

$$\textit{Demand}: \quad \sum_{r \in S_{i,j}} x^3_{r,j,(t-\Delta^{n,j}_r)} = D^3_{i,j,t}(\omega) \quad \forall i, j, t$$

Algebraic Formulation (cont.)

Joint allocation

Flying times :

$$\sum_{t',O(r)=l} T^m_{j,r,t,t'} x^m_{r,j,t'} \leq A_{j,t}(\omega)(y_{j,l,m,t} + \sum_{\{p:p\in AV_{t'},j_p=j,l_p=l\}} I_{\hat{f},p,t}) \quad \forall j \in G_1, l, m, t$$

$$\sum_{t',O(r)=l} T^m_{j,r,t,t'} x^m_{r,j,t'} - A_{j,t}\hat{y}^2_{j,l,m,t} \leq 0 \quad \forall j \in G_2, l, m, t$$

$$\sum_t \sum_r T'^m_{r,j}(x^m_{r,j,t}) \leq \sum_t \mu_j(y_{j,l,m,t}) + \sum_t \sum_{\{p:p\in AV_{t'},j_p=j,l_p=l\}} \mu I_{\hat{f},p,t} \quad \forall j \in G_1, l, m$$

Air Refueling :

$$\sum_{r\in Q1_l} \sum_j TR_{r,j,l} x^m_{r,j,(t-\Delta^j_r)} = \sum_{r\in Q2_l} \sum_{j\in JT} x^m_{r,j,t} \quad \forall l \in LA, t, m$$