Split-and-Merge Method for Accelerating Convergence of Stochastic Linear Programs

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MOTIVATION

US Air Mobility Command Mission: Optimally schedule 1300 aircraft for
Cargo shipment
Personnel movement
Distinguished visitor support
Air refueling

CHALLENGE

Mission requirements subject to
demand delays
demand changes
aircraft breakdown
weather events
natural disasters
conflicts

STOCHASTIC OPTIMIZATION OVERVIEW

Stage 1 (strategic decision)
Cost of resource allocation +
Expected cost of stage 2 decisions
(Integer Linear Program)
Planning Stage: Makes
decision about known parameters
Execution stage: Assumes
probabilistic distribution of
uncertain parameters
Known and evaluates them
for the stage 1 decision

Stage 2 (operational decisions)
Execution cost
(Integer Linear Program)

APPLICATIONS

STATE-OF-THE-ART CANNOT SOLVE LARGE
PROBLEMS TO OPTIMALITY

What is new that we are doing?
Combine stochastic programming with high performance computing
Facilitates reconciliation of myriad possible outcomes in a timely manner
Makes it feasible to solve integer programs

CASE STUDY: US MILITARY AIRCRAFT ALLOCATION PROBLEM

US AMC yearly expenses: *USD 4 Billion
Average cost benefit: 35%
Robustness: 66% reduction in variance

RELATED WORK

- Magnanti and Wong, 1981
- Add only dominating cuts
- Requires solving additional optimization problems
- Linderoth et al., 2003
- Requires solving additional optimization problems to determine usability of cuts
- Trust Region, Ruszczynski, 1886 and Linderoth et al., 2003
- Add objective term to minimize volume of candidate solution
- Requires doing several minor iterations between major iterations
- Progressive Hedging Algorithm, 1991
- Requires search for optimal Lagrangean multiplier which can be prohibitive

PROPOSED SPLIT-AND-MERGE (SAM) METHOD

Split original problem into many small
subproblems each with a subset of scenarios

Perform stochastic linear optimization
of subproblems (in parallel) until
converged or until a specified number
of iterations

NESTED BRANCH-AND-BOUND PARALLELISM

Relax integer program to linear program
Each vertex is a stochastic linear program

RESULTS

Multicut Benders method
- 34% improvement in time
- Total iterations = 415
Time to Solution = 784s

SAM method
- 58% improvement in time
- Total iterations = 360
Time to Solution = 507s

SIPs Performance
1.96x speedup from 3 to 480 cores

Gurobi IP Solver Performance
Typical parallel efficiency of just 0.1 at 32 cores

Challenge
Root vertex solve is a bottleneck, when #scenarios are large

REFERENCES

