Enabling Massive Parallelism for Stochastic Optimization Problems

[Extended Abstract]

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ABSTRACT

The US air fleet is tasked with the worldwide movement of cargo and personnel. Due to a unique mixture of operating circumstances, it faces a large scale and dynamic set of cargo movement demands with sudden changes almost being the norm. Airfleet management involves periodically allocating aircraft to its myriad operations, while judiciously accounting for this uncertainty to minimize operating costs. We have formulated this allocation problem as the optimization of a stochastic two-stage integer program.

Our work aims to enable rapid decisions via a scalable parallel implementation. We present our initial attempts at parallelization and eventually, a branch-and-bound approach with two-stage linear programs. This allows the evaluation of tens of thousands of possible scenarios while converging to an optimal integer allocation for extremely large problems. We believe that this is an interesting and uncommon approach to harnessing tera/petascale compute power for such problems without decomposing the linear programs further.

Categories and Subject Descriptors

G.1.6 [**Mathematics of Computing**]: Numerical Analysis—*Optimization: Stochastic Programming*; I.6 [**Simulation and Modeling**]: Applications—*airfleet management*; D.1.3 [**Software**]: Programming Techniques—*Parallel Programming*; J.1 [**Computer Applications**]: Administrative Data Processing—*Military*

General Terms

Algorithms, Design, Performance

Keywords

Stochastic optimization, Parallel Branch and Bound, Simulation, Airfleet Management

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1. INTRODUCTION

DoD manages a fleet of over 1300 aircrafts that operate globally, frequently with little notice, to destinations lacking infrastructures but populated with hostile forces. Aircraft are allocated by the military at its different bases in anticipation of the demands for several missions to be conducted in the period of one month. Aircraft breakdowns, weather, natural disaster, conflict, are some of the various possible outcomes that confound decision support. The purpose of the stochastic model is to optimally allocate aircraft against each mission area in a manner that minimizes subsequent disruption. We model the allocation process as a two-stage stochastic mixed-integer program with complete recourse. Aircraft are allocated in the first stage; the second stage subproblems conduct more detailed planning with probability-weighted mission and cargo demands over hundreds of scenarios.

We start with a conventional master-worker based parallel decomposition of the problema and analyze the challenges involved. Since the master involves solving an Integer program in every iteration, it acts as a huge sequential bottleneck significantly affecting the parallel efficiency and also making reaal-worls problems intractable. We address this problem by using a parallel branch and bound based design that converges to an optimal integer allocation for large problems while al- lowing evaluation of tens of thousands of possible scenarios.

2. MASTER WORKER PARALLELIZATION

As Figure 1 shows that master acts as a serious bottleneck, and hence this approach does not scale well to large number of cores. Moreover, with just 120 scenarios the program cannot exploit parallelism from more than 122 cores (1 master + 120 workers + 1 coordinator).

Figure 2 shows the scalability plot for a 10 time-period model (with stage 1 as a linear program) of the aircraft allocation problem.

3. PARALLEL BRANCH AND BOUND

We propose parallelizing stage 1 by solving linear program in stage 1 and branching on fractional variables until an inte-

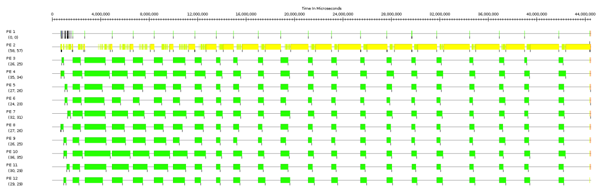

Figure 1: Projection timelines for master worker implementation

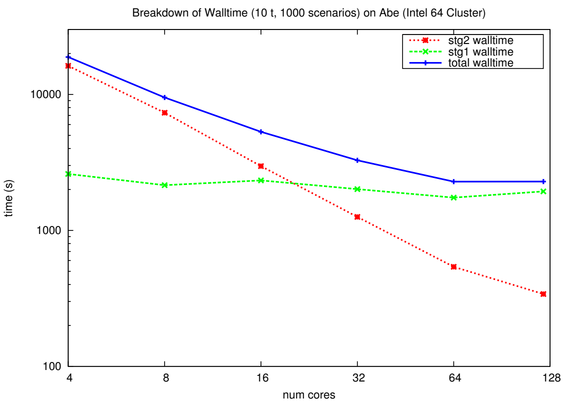

Figure 2: Scalability plot of the master worker implementation with 120 scenarios in stage 2

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ger allocation is obtained or the vertex gets pruned because of the lower bound exceeding the current best incumbent cost. Thus we can exploit two sources of parallelism:

- Scenario parallelization i.e. the stage 2 parallelization
- Parallel optimization of the branch and bound tree vertices

A novel branch-and-bound based design is presented that uses scenario clustering in stage 2, cut-sharing amongst the tree vertices in stage1 and advanced(warm) start provided by the linear program solvers for performance optimization. This approach is massively parallel and can be used to solve large-sized stochastic integer programs. Figure 3 compares the projection timelines of the Branch and Bound based approach with the master worker parallelization. Scalability plot of this new approach is presented in the poster.

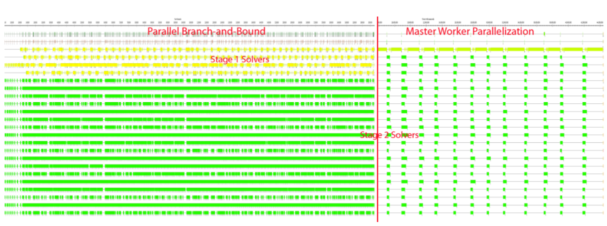

Figure 3: Comparing the projection timelines of the new Branch and Bound based approach with the conventional Master Worker Parallelization

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APPENDIX

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